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An adaptive subspace trust-region method for frequency-domain seismic full waveform inversion

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ABSTRACT

Full waveform inversion is currently considered as a promising seismic imaging method to obtain high-resolution and quantitative images of the subsurface. It is a nonlinear ill-posed inverse problem, the main difficulty of which that prevents the full waveform inversion from widespread applying to real data is the sensitivity to incorrect initial models and noisy data. Local optimization theories including Newton's method and gradient method always lead the convergence to local minima, while global optimization algorithms such as simulated annealing are computationally costly. To confront this issue, in this paper we investigate the possibility of applying the trust-region method to the full waveform inversion problem. Different from line search methods, trust-region methods force the new trial step within a certain neighborhood of the current iterate point. Theoretically, the trust-region methods are reliable and robust, and they have very strong convergence properties. The capability of this inversion technique is tested with the synthetic Marmousi velocity model and the SEG/EAGE Salt model. Numerical examples demonstrate that the adaptive subspace trust-region method can provide solutions closer to the global minima compared to the conventional Approximate Hessian approach and the L-BFGS method with a higher convergence rate. In addition, the match between the inverted model and the true model is still excellent even when the initial model deviates far from the true model. Inversion results with noisy data also exhibit the remarkable capability of the adaptive subspace trust-region method for low signal-to-noise data inversions. Promising numerical results suggest this adaptive subspace trust-region method is suitable for full waveform inversion, as it has stronger convergence and higher convergence rate.

1. Introduction

As a recent alternative method for physics model building, full waveform inversion (FWI) allows to reconstruct high-resolution models of the subsurface through the extraction of full information contained in the seismograms. Compared with traveltime tomography, whose resolution is the width of the first Fresnel zone, full waveform inversion has a surprisingly high resolution of the order of the wavelength (Pratt et al., 1996). Apart from this, another main advantage of the full waveform inversion is the extensive application to different acquisition geometries and prospecting scales. Results with real crosshole data have been demonstrated by Song et al. (1995). Operto et al. (2004) and Ravaut et al. (2004) applied waveform inversion to onshore wide-angle data. Since the first attempt in the 1980s, many researchers have become interested in the potential benefits of the high resolution and no limits of the acquisition geometries of the full waveform inversion. In addition, FWI can be viewed as an automated process and can potentially fill the traditional gap of resolution between macro-velocity model building and migration. During the last decade, it was widespread applied in exploration (Virieux and Operto, 2009), shallow environmental investigation (Gao et al., 2007), macroscale velocity modeling and imaging (Ben-Hadj-Ali et al., 2008) and the research of the structure of Earth's interior (Operto et al., 2006).

Waveform inversion was originally developed in the time-space domain. At first, its application in research was handicapped by great cost in terms of computing time. Then, Tarantola (1984) showed that the gradient could be obtained without computing the derivative explicitly, which substantially reduced the computational cost and conduced to a great number of applications in real data (More, 1988). In the 1990s, Pratt and Worthington (1990) and Pratt (1990) researched the implementation of full waveform

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inversion in frequency domain with acoustic equation and elastic equation respectively. Using a limited number of frequency data components in their inversion, it significantly reduced the scale of the problem and improved the computational efficiency. Sirgue and Pratt (2004) provided the theoretical basis for selecting the frequencies, indicating that an unaliased image could be obtained with these selected frequencies. Until recently, Shin and Cha (2008) completed the Laplace domain full waveform inversion, which can recover main structures of the subsurface model even with the observational data sets lacking low-frequency information, the inverted model is always used as an initial model for frequency domain inversion. The full waveform inversion in this paper is implemented in the frequency domain due to its impressive numerical advantages when a large number of source locations are involved.

Full waveform inversion aims to recover the true model by iteratively minimizing the difference between observed and modeled data, which is a nonlinear ill-posed inverse problem with many solutions. As to such kind of problem, the solution is sensitive to the noise in data sets and inaccurate starting model. So far gradient method and Newton's method have been the main optimization algorithms in use thanks to their fast convergence to an acceptable solution. However, this solution may be a local minimum of the objective function that produces spurious results. To mitigate the local minima, several researchers made great efforts and achieved considerable improvements (Virieux and Operto, 2009; Asnaashari et al., 2013; van Leeuwen and Herrmann, 2013). Global optimization algorithms such as simulated annealing (SA) (Sen and Stoffa, 1991), genetic algorithms (GA) (Tran and Hiltunen, 2012) and particle swarm optimization algorithm (PSO) (Zhu et al., 2011) have already been used in full waveform inversion. Global optimization schemes have advantages over local optimization approaches in that the former can utilize the nonlinear of the inversion problem to find the global minima (Shin and Ha, 2008). However, these global techniques are computationally practical only with small scale models or simple models because of their blindness search and enormous times of the forward modeling, hence unacceptable for large scale models. Sen and Stoffa (1991) performed $\mathbf{M} \times \mathbf{N}$ forward calculations in each iteration, where $\mathbf{N}$ referred to the number of model parameters and $\mathbf{M}$ meant all the possible values that one model parameter could take, with at least 30 iterations, sometimes 100 iterations needed to obtain a high crosscorrelation value between observed data and modeled data, which results in its applicability to only 1-D models with fewer than 10 parameters. Similarly, Tran and Hiltunen (2012) employed genetic algorithms to 1-D waveform inversion with only one source.

The trust-region method developed by Powell (1970) has proved to be more efficient and has better convergent property than line search methods, especially for ill-posed inverse problems (Wang and Yuan, 2005). Many trust-region algorithms have a potential to converge to the global minimum under certain conditions (Powell, 1984). In trust-region method, an approximate model will be constructed near the current iterate point and the solution of this approximate model will be taken as the next iterate point. Compared to the line search algorithms, the trust-region method only trust the approximate model in a region near the current iterate. This is reasonable, because for general nonlinear functions local approximate models can only fit the original function locally. The region that the approximate model is trusted is called the trust region. Due to its strong convergence properties and robustness, trust-region methods have been studied by many researchers and applied in many disciplines including geophysical inversion. Zeng et al. (2011) introduced the trust-region algorithm into magnetotelluric data inversion and obtained a good result. Tian and Chen (2006) combined the quasi-Newton method and the trust-region method to determine hypocenters and velocity structures, proving that it is more accurate and efficient and has better convergent property than the quasi-Newton method. Li and Wang (2012) applied the trust-region method to seismic migration inversion. All these applications substantiate its efficiency for the ill-posed geophysical inversion problem.

Compared to line search methods such as gradient method and Newton's method, the trust-region method can mitigate local minimum and converge to a better solution; while compared to global algorithms, such as GA, the trust-region method is less computationally intensive and requires less memory storage, especially when some special trust-region methods for large scale problems occur. In this paper we investigate the possibility of applying the trust-region method to the full waveform inversion problem. Thanks to the large scale of the full waveform inversion, we implement the subspace trust-region method proposed by Wang and Yuan (2006) to reduce the computations. A key content of a trust-region method is how to update the trust region radius. We study a new adaptive trust-region method in which we consider not only factors such as gradient, but also the specificity of full waveform inversion to update the trust-region radius. Then we introduce this adaptive trust-region method into the frequency domain full waveform inversion and any reference on similar research has not been found by far.

The paper is divided into three main sections. In the first section, we briefly present the theory of full waveform inversion in the frequency domain. We then proceed to detail the fundamental and workflow of the adaptive subspace trust-region method. In the third and final section of this paper, numerical examples with synthetic data are performed to illustrate the potential of the developed method for providing high-resolution subsurface models.

2. Theory

The theory of frequency-domain waveform modeling and inversion used in this paper are proposed by Jo et al. (1996), Stekl and Pratt (1998), Hustedt et al. (2004), Pratt and Worthington (1990) and Pratt (1990), we will here briefly review the methodology.

2.1. Frequency-domain forward modeling

Frequency-domain visco-acoustic wave equation is expressed as

$$\frac{\omega^2}{\kappa(x, z)}p(x, z, \omega) + \frac{\partial}{\partial x}\left(\frac{1}{\rho(x, z)}\frac{\partial p(x, z, \omega)}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{1}{\rho(x, z)}\frac{\partial p(x, z, \omega)}{\partial z}\right) = -s(x, z, \omega) \tag{1}$$

where $\rho(x, z)$ is the density, $\kappa(x, z)$ is the complex bulk modulus, ω is the angular frequency, $p(x, z, \omega)$ is the pressure field and $s(x, z, \omega)$ is the source. Attenuation can be implemented in Eq. (1) by using complex velocities.

Using the finite-difference method, the visco-acoustic wave equation, Eq. (1), can be recast in a matrix form as

$$\mathbf{A}\mathbf{p} = \mathbf{s} \tag{2}$$

where $\mathbf{A}$ is the complex impedance matrix depending on frequency and properties of the medium, $\mathbf{p}$ and $\mathbf{s}$ are vectors of dimension $l \times 1$ denoted by $l = nx \times nz$, where nx and nz are the grid size of the model in horizontal and vertical directions, respectively. The pressure field $\mathbf{p}$ is obtained by solving the system, Eq. (2), with