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A mesh-free method with arbitrary-order accuracy for acoustic wave propagation

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ABSTRACT

In the present study, we applied a novel mesh-free method to solve acoustic wave equation. Although the conventional finite difference methods determine the coefficients of its operator based on the regular grid alignment, the mesh-free method is not restricted to regular arrangements of calculation points. We derive the mesh-free approach using the multivariable Taylor expansion. The methodology can use arbitrary-order accuracy scheme in space by expanding the influence domain which controls the number of neighboring calculation points. The unique point of the method is that the approach calculates the approximation of derivatives using the differences of spatial variables without parameters as e.g. the weighting functions, basis functions. Dispersion analysis using a plane wave reveals that the choice of the higher-order scheme improves the dispersion property of the method although the scheme for the irregular distribution of the calculation points is more dispersive than that of the regular alignment. In numerical experiments, a model of irregular distribution of the calculation points reproduces acoustic wave propagation in a homogeneous medium same as that of a regular lattice. In an inhomogeneous model which includes low velocity anomalies, partially fine arrangement improves the effectiveness of computational cost without suffering from accuracy reduction. Our result indicates that the method would provide accurate and efficient solutions for acoustic wave propagation using adaptive distribution of the calculation points.

1. Introduction

Forward modeling techniques of wave propagation are indispensable tools for the implementation of reverse time migration (RTM) and full waveform inversion (FWI) (Tarantola, 1984; Virieux et al., 2011). Recently, more complex models like salt dome model (e.g. BP model), which include large velocity contrasts, become a target of FWI (Cha and Shin, 2010). Since the most computationally expensive part of these numerical schemes is the forward modeling, the computational efficiency is recognized as one of the keys to improve the effectiveness of the schemes. For the calculation of full-waveform synthetic seismic traces in inhomogeneous models, numerical simulation methods such as finite difference (FD) and finite element (FE) have often been used. FD method has been widely used for many years as a simulator of acoustic wave propagation, and highly accurate and efficient FD operators developed by many researchers (e.g. Virieux, 1986; Chu and Stoffa, 2012; Liu et al., in press, 2014; Tan and Huang, 2014) are available. These schemes are compared to each other in terms of the numerical accuracy and computational efficiency (e.g. Liang et al., in press). In many cases, the coefficients of FD operators are derived based on the regular lattice grids. To overcome problems that may arise to handle arbitrary shaped anomalies or topographies using the regular lattice grids, curvilinear schemes for modeling wave propagation have been developed (e.g. Tarrass et al., 2011). Although these schemes can handle arbitrary shaped topography, arrangement of optimal grid for complex velocity models is not straightforward. On the other hand, FE method uses numerical meshes to build arbitrary shaped models. The method provides the flexibility and the accuracy in the calculation through the mesh generation process, which is computationally costly. It is meaningful to have other methods that could deal with non-flat surface or interfaces with less computational load than FE method.

Some novel approaches based on a mesh-free concept have also been developed. This class of numerical methods can discretize models of analysis, which include complex topography and/or complex velocity structure, without any mesh structure or regular lattice grids, and use a set of calculation points surrounding each target point for the discretization (e.g. Lee et al., 2003). Wittke and Tezkan (2014) presented a new approach for magnetotelluric modeling using the Meshless Local Petrov–Galerkin method. Wenterodt and Estorff (2009) investigated the dispersion

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property of the meshfree radial point interpolation method (RPIM) for the Helmholtz equation, and showed a significant reduction of the dispersion error compared with the FE method. The method, however, requires background meshes to conduct the numerical integration. Furthermore, we need to define not only the radius of the influence domain but also the weighting and basis functions. These miscellaneous parameters lead to the complexity in the choice of optimal combination for minimizing the dispersion error (Wenterodt and Estorff, 2011).

O'Brien and Bean (2011) developed an irregular lattice method for elastic wave propagation based on an elastic lattice method (Monette and Anderson, 1994; Toomey and Bean, 2000; O'Brien and Bean, 2004). They overcame the restrictions on the regular lattice through the augmentation in the number of the nearest neighbor points. Takekawa et al. (2012) proposed a particle method to simulate seismic wave propagation induced by earthquakes. The method can introduce free-surface condition just by removing or ignoring any particles above the surfaces, and could be applied to computational rock physics problems (Takekawa et al., 2014a). These methods do not require the background meshes for the numerical integration, and could be classified as true mesh-free methods. However, the methods do not improve the order of the accuracy in space even if the number of neighbors is increased (Takekawa et al., 2014b, 2014c). For the utilization of mesh-free models in the forward simulation, the accuracy of methods to apply to the models needs to be revisited.

In this study, we present a mesh-free method for solving acoustic wave propagation that could provide the accuracy of arbitrary order based on the multivariable Taylor expansion (Tamai et al., 2013). The method was originally developed for solving incompressible fluid flow with the free surface, and provided arbitrary-order accuracy in space (Tamai et al., 2013). The high-order scheme could be applied to irregular distributions of particles successfully without any background meshes, i.e. it is also a true mesh-free method. Since the method was originally designed as a general method for solving partial differential equations, we are able to extend the method to solve the acoustic wave equation. The feature of the method is that the approximation of derivatives is calculated by using the differences of spatial variables without parameters as e.g. the weighting functions, basis functions. In other words, the method is a mesh-free FD method. This feature of the method eliminates the complicated process of parameter optimization (Wenterodt and Estorff, 2011).

In the present study, we first introduce the basic concept of the method followed by the verification of the dispersion property for both regular and irregular arrangements of calculation points. We then calculate acoustic wave propagation using a homogeneous model with random distribution of calculation points. Finally, we demonstrate the effectiveness of the method using an inhomogeneous model and confirm that our method would be a true mesh-free method where the accuracy can be quantitatively measured.

2. Method

In this section, we explain the basic concept of the mesh-free method based on the multivariable Taylor expansion. The multivariable Taylor expansion of a scalar function $f(\mathbf{r})$ to Mth order at position $\mathbf{r}_i$ is expressed as follows:

$$\begin{aligned} f(\mathbf{r}_i+\Delta\mathbf{r}) = f(\mathbf{r}_i) &+ \frac{1}{1!}\left(\Delta r_1\frac{\partial}{\partial r_1}+\cdots+\Delta\mathbf{r}_d\frac{\partial}{\partial\mathbf{r}_d}\right)f(\mathbf{r}_i) \\ &+ \frac{1}{2!}\left(\Delta r_1\frac{\partial}{\partial\mathbf{r}_1}+\cdots+\Delta\mathbf{r}_d\frac{\partial}{\partial\mathbf{r}_d}\right)^2 f(\mathbf{r}_i)+\cdots \\ &+ \frac{1}{M!}\left(\Delta\mathbf{r}_1\frac{\partial}{\partial\mathbf{r}_1}+\cdots+\Delta\mathbf{r}_d\frac{\partial}{\partial\mathbf{r}_d}\right)^M f(\mathbf{r}_i)+O\left(\|\Delta\mathbf{r}\|^{M+1}\right) \end{aligned} \tag{1}$$

where $\mathbf{r}_i$ and $\mathbf{r}_i+\Delta\mathbf{r}$ are the position vectors of calculation point i and its neighboring point j, d is the number of spatial dimension. In many cases related to wave propagation, d may be 2 or 3. $\Delta\mathbf{r}$ is the relative position vector between points i and j. $\Delta\mathbf{r}_d$ means dth component of vector $\Delta\mathbf{r}$ (in two-dimensional case, $\Delta\mathbf{r}=(\Delta\mathbf{r}_1, \Delta\mathbf{r}_2)$).. We replace $f(\mathbf{r}_i)$ and $f(\mathbf{r}_i+\Delta\mathbf{r})$ into $f_i\ (=f(\mathbf{r}_i))$ and $f_j\ \left(=f(\mathbf{r}_j)=f(\mathbf{r}_i+\Delta\mathbf{r})\right)$, respectively.

Here, we define vectors $\mathbf{P}$ and δ as follows:

$$\mathbf{P}=\left(\Delta\mathbf{r}_1,\ \ldots,\ \Delta\mathbf{r}_d,\ \frac{1}{2!}\Delta\mathbf{r}_1^2,\quad \Delta\mathbf{r}_1\Delta\mathbf{r}_2,\ \ldots,\ \frac{1}{2!}\Delta\mathbf{r}_d^2,\ \ldots\right.$$
$$\left.,\ \frac{1}{(M-1)!1!}\Delta\mathbf{r}_{d-1}\Delta\mathbf{r}_d^{M-1},\quad \frac{1}{M!}\Delta\mathbf{r}_d^M\right)^T \tag{2}$$

$$\boldsymbol{\delta}=\left(\frac{\partial}{\partial\mathbf{r}_1},\ \ldots,\ \frac{\partial}{\partial\mathbf{r}_d},\ \frac{\partial^2}{\partial\mathbf{r}_1^2},\ \frac{\partial^2}{\partial\mathbf{r}_1\partial\mathbf{r}_2},\ \ldots,\ \frac{\partial^2}{\partial\mathbf{r}_d^2},\ \ldots,\ \frac{\partial^M}{\partial\mathbf{r}_{d-1}\partial\mathbf{r}_d^{M-1}},\ \frac{\partial^M}{\partial\mathbf{r}_d^M}\right)^T \tag{3}$$

Vectors $\mathbf{P}$ and δ include coefficients and derivatives, respectively. Using Eqs. (2) and (3), we transform Eq. (1) as follows:

$$\{(\mathbf{P}\cdot\boldsymbol{\delta})f\}_{\mathbf{r}=\mathbf{r}_i}=\Delta f_{ij}+O\left(\Delta\|\mathbf{r}\|^{M+1}\right) \tag{4}$$

where $\Delta f_{ij}=f_j-f_i$. Multiplying both sides of Eq. (4) by $\mathbf{P}$, we obtain

$$\begin{aligned} &\{(\mathbf{P}\cdot\boldsymbol{\delta})(\mathbf{P}f)\}_{\mathbf{r}=\mathbf{r}_i}=\mathbf{P}\Delta f_{ij}+\mathbf{P}\cdot O\left(\|\Delta\mathbf{r}\|^{M+1}\right) \\ &\Rightarrow(\mathbf{P}\otimes\mathbf{P})\cdot(\boldsymbol{\delta}f)_{\mathbf{r}=\mathbf{r}_i}=\mathbf{P}\Delta f_{ij}+\mathbf{P}\cdot O\left(\|\Delta\mathbf{r}\|^{M+1}\right) \end{aligned} \tag{5}$$

where $\mathbf{a}\otimes\mathbf{b}$ means the tensor product of vectors $\mathbf{a}$ and $\mathbf{b}$.

Here, we introduce an influence domain which supports a finite region around $\mathbf{r}=\mathbf{r}_i$. This domain controls the number of neighboring calculation points. Wider range of the influence domain contains larger number of calculation points. The effect of the influence domain on the accuracy and calculation time is investigated in the following section. We calculate the sum of both sides of Eq. (5) inside the influence domain.

$$\left\{\sum_j^n(\mathbf{P}\otimes\mathbf{P})\right\}\cdot(\boldsymbol{\delta}f)_{\mathbf{r}=\mathbf{r}_i}=\sum_j^n\left(\mathbf{P}\Delta f_{ij}\right)+\sum_j^n\mathbf{P}\cdot O\left(\|\Delta\mathbf{r}\|^{M+1}\right) \tag{6}$$

$$(\boldsymbol{\delta}f)_{\mathbf{r}=\mathbf{r}_i}\approx\left\{\sum_j^n(\mathbf{P}\otimes\mathbf{P})\right\}^{-1}\cdot\left\{\sum_j^n\left(\mathbf{P}\Delta f_{ij}\right)\right\} \tag{7}$$

n is the number of neighboring calculation points inside the influence domain. Vector (δf) in Eq. (7) includes derivatives of $f(\mathbf{r})$ at $\mathbf{r}=\mathbf{r}_i$. The size of the matrix $\mathbf{P}\otimes\mathbf{P}$ depends only on the order of accuracy M. For example, in two-dimensional case, the size are 5×5 and 14×14 for $M=2$ and 4, respectively. Since the value of each component of the matrix depends only on the relative positions between point i and neighboring points j, the inverse of the matrix can be calculated before starting time steps. Once the inverse at each calculation point is fixed, we continue to use it during the calculation. This means that solving inverse matrices, which is a challenging procedure, can be excluded from the time loop.