Finance Research Letters 22 (2017) 95–100

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Implementing and testing the Maximum Drawdown at Risk

Beatriz Vaz de Melo Mendes[a,*], Rafael Coelho Lavrado[b]

[a] *COPPEAD/IM, Federal University at Rio de Janeiro, Brazil*
[b] *IMPA, Instituto Nacional de Matemática Pura e Aplicada, Brazil*

ARTICLE INFO

Article history:
Received 22 October 2016
Revised 26 May 2017
Accepted 3 June 2017
Available online 9 June 2017

JEL classification:
C14
C15
C53
C58
G32

Keywords:
Risk management
Maximum drawdown
ARMA-GARCH
Simulations

ABSTRACT

Financial managers are mainly concerned about long lasting accumulated large losses which may lead to massive money withdrawals. To assess this risk feeling we compute the Maximum Drawdown, the largest price loss of an investment during some fixed time period. The Maximum Drawdown at Risk has become an important risk measure for commodity trading advisors, hedge funds managers, and regulators. In this study we propose an estimation methodology based on Monte Carlo simulations and empirically validate the procedure using international stock indices. We find that this tool provides more accurate market risk control and may be used to manage portfolio exposure, being useful to practitioners and financial analysts.

1. Introduction

Financial managers are mainly concerned about large losses because they may destroy accumulated wealth leading to massive money withdrawals and risking the continuity of businesses. Financial crises such as the 2008 global one have shown how extensive these losses could be, with investment funds all over the world showing more than 50% losses and survivors taking several years to recover. Investors have now become more cautious, trying hard to avoid large negative portfolio changes.

Given a fixed time period, the *Maximum Drawdown* (MDD) may be defined as the largest percentage loss of an investment over this period. Following an extremely large fall (or a long sequence of small falls) in market prices, an investor (specially retirees) may decide to sell valuable positions irrespective of market conditions for fear of even larger losses. Tracking the drawdown helps controlling the risk and preserving the capital of an investment.

To manage risk several risk measures are available capturing various aspects of risk, the most popular one being the Value at Risk (VaR). The *Maximum Drawdown at Risk* (MDaR), defined as a percentile of the MDD distribution, has become an important and useful tool for hedge funds managers, commodity trading advisors, and regulators.

In the literature, one will find few but important related works. Cvitanić and Karatzas (1999) study the MDD as a risk measure. Chekhlov et al. (2000) define the Conditional Expected Drawdown (CDaR) as the mean of all drawdowns exceeding

* Corresponding author at: Rua Ministro Raul Fernandes 210, apt. 1107,Rio de Janeiro 22260040, RJ, Brazil.
E-mail address: beatriz@im.ufrj.br (B.V.d.M. Mendes).

http://dx.doi.org/10.1016/j.frl.2017.06.001
1544-6123/

a particular drawdown level. Mendes and Brandi (2004) compute parametric estimates of the CDaR by fitting the Modified Generalized Pareto Distribution and its sub-models to the extreme tail of drawdowns.

Hoesli and Hamelink (2004) show that most portfolios optimized in the return-MDD space yield a lower MDD than those based on the popular mean-variance approach. Rebonato and Gaspari (2006) and Pospisil and Vecer (2008) study the statistical properties of the MDD. Pospisil and Vecer (2010) define new sensitivities as the derivatives of a financial contract value with respect to the MDD. Kim (2011) links the mean-variance analysis of Markowitz and the MDD, explaining the role of MDD in the investment fund selection problem.

Following Chekhlov et al. (2000) definition, Goldberg and Mahmoud (2014) show that the Conditional Expected Drawdown (CED) is not a coherent risk measure but a convex measure, and hence can be used as an optimizer. Portfolio optimization using drawdowns has also been considered in Chekhlov et al. (2000). Other related studies include Grossman and Zhou (1993), Harding et al. (2003), Leal and Mendes (2005), Hayes (2006), Vecer (2007), and Gray and Vogel (2013).

The contribution of this paper to the existing literature is three-fold. First, an accurate semiparametric estimation methodology for the MDaR is developed in easy steps. We fit an econometric model to the data (parametric step) and estimate the risk measure through simulations (nonparametric step). Second, the proposed estimation approach is tested using eight important stock indices. We assess the performance of the MDaR estimates by computing the observed and expected number of threshold violations and applying a formal test and carrying on sensitivity analyses on model assumptions. The model-based estimates of the MDaR are found very accurate and proved to respond quickly to changes in the volatility level. Third, we show the MDaR usefulness as a tool in a dynamic investment strategy, with the resulting portfolio presenting lower risk (smaller volatility) and smaller drawdowns.

The remainder of this paper proceeds as follows. Sections 2 and 3 formally define and discuss the methodologies for computing the MDD. Section 4 describes the data and perform the empirical analyses, evaluating the proposed methodology through tests. Section 5 summarizes the results.

2. Risk measures

A risk measure quantifies in just one number the risk of a portfolio, and to assess tail risk one needs risk measures such as the MDaR derived from extreme statistics. Let $p_t = ln(P_t)$ be the logarithm of the asset price P_t at time t, $t \in \{1, \cdots, H\}$. The MDD during this period may be defined as

$$MDD = \max_{1 \le k < H} \max_{k < j \le H} \{p_k - p_j, 0\}. \quad (1)$$

This definition yields a non-negative random variable whose duration D, $1 \le D = j - k < H$, the length of the sequence of log-prices, is also a random variable. When $D = 1$, the MDD coincides with the worst single (one-period) loss within the window, the Maximum Loss. Alternatively, the MDD may be defined in percentage terms:

$$MDD = \max_{1 \le k < j \le H} \left(\frac{P_k - P_j}{P_k} \right), \quad (2)$$

$1 \le k < j \le H$, where H is the window size, or zero if all $P_k \le P_j$.

The MDD being defined on a sequence of prices (or returns) is affected by the strength of serial dependence shown by the returns, and its magnitude is sensitive to the crucial choice of H.

Fig. 1 shows the MDD behavior using a SP&500 two-year sample (505 observations) from January/2008 through December/2009. The first row shows the evolution over time of the log-prices and log-returns, clearly indicating the crisis effect. The second row shows the time series plot of the MDD (formulas (1) and (2)) based on a daily shifted window with $H = 22$ days. The third row shows the empirical distributions of the MDD size and duration. Among the 483 MDDs there are thirty impressive long durations: seven of 21 days, nine of 20 days, and fourteen MDDs lasting for 19 days.

The Maximum Drawdown at Risk α ($MDaR_\alpha$) is defined as the $(1 - \alpha)$-quantile of the MDD distribution. While the VaR_α is usually computed for short-time horizons, usually one or five days, the $MDaR_\alpha$ is preferably used for longer horizons, at least 10 days.

3. Methodologies for estimating the MDar$_\alpha$

Models for the future distribution of either the MDD or the returns are chosen from three large classes: parametric, non-parametric, and semi-parametric.

Econometric models (parametric). The simplest approach is to fit a parametric distribution to the data and compute the desired quantile. Appropriate distributions the for MDD, designed to model extremes and capture tail characteristics, come from the Extreme Value Theory. More sophisticated conditional models will be considered under the semiparametric approach.

Historical simulation (nonparametric). This approach makes no assumptions on the data generating process and apply the empirical distribution to estimate the unconditional underlying distribution. The historical $MDaR_\alpha$ for a period of H days is the $(1 - \alpha)$% empirical percentile of the MDD series. Fig. 1 illustrates and shows the historical $MDaR_{5\%}$ (28.58%). We observe a right long tail which is indeed a stylized fact about the MDD distribution.