Finance Research Letters 000 (2017) 1–6

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Fast fractional differencing in modeling long memory of conditional variance for high-frequency data

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ARTICLE INFO

Article history:
Received 28 December 2016
Accepted 30 December 2016
Available online xxx

JEL classification:
C22
C51
C53
C58

Keywords:
Computation time
Fractional integration
Fourier transforms
GARCH
Long memory
High-frequency data

ABSTRACT

We transfer the recently introduced fast fractional differencing that utilizes fast Fourier transforms (FFT) to long memory variance models and show that this approach offers immense computation speedups. We demonstrate how calculation times of parameter estimations benefit from this new approach without changing the estimation procedure. A more precise depiction of long memory behavior becomes feasible. The FFT offers a computational advantage to all ARCH(∞)-representations of widely-used long memory models like FIGARCH. Risk management applications like rolling-window Value-at-Risk predictions are substantially sped up. This new approach allows to calculate the conditional volatility of high-frequency in a practicable amount of time.

1. Introduction

In time series analysis, long range dependence has been focused on for a long time. Hosking (1981) identifies the lack of a proper model formulation for the long memory property and extends the Autoregressive Integrated Moving Average (Box et al. 2008) by allowing for a real valued degree of differencing. Henceforth, this characterization is referred to as fractional differencing. This new family of processes are the widely used Autoregressive Fractionally Integrated Moving Average (ARFIMA) class. Around the same time, GARCH processes for modeling the conditional variance of time series are introduced (Engle, 1982; Bollerslev, 1986). While these models have an exponentially declining memory which is considered 'short memory', some applications require longer persistence of shocks. The idea of fractional differencing is carried over to variance modeling with the introduction of FIGARCH (Baillie et al., 1996). Applying the FIGARCH model, a vast amount of literature shows the long range dependence of distant observations in many different financial time series, e.g. equity, foreign exchange, and commodity markets (e.g. Tse, 1998; Tang and Shieh, 2006; Mensi et al., 2014). Long memory volatility models are used in various risk management applications, where forecasting of volatility is focused on.

Stoev and Taqqu (2004) and Jensen and Nielsen (2014) present a fast fractional differencing algorithm for ARFIMA models which is based on the fast Fourier transform (FFT) algorithm by Cooley and Tukey (1965). Inspired by this idea, we transfer

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http://dx.doi.org/10.1016/j.frl.2016.12.020
1544-6123/

this methodology to long memory volatility models. For these models, it is shown that the FFT approach offers tremendous speedups compared to traditional convolution summations and might yield better estimation results than naive summations by allowing for greater truncation lags. These findings are of great importance for practitioners and research focusing on the long memory property of conditional variance of time series. Moreover, this approach eventually allows to examine the structure of conditional variance of high-frequency data in a feasible amount of time.

The remainder of this article is structured as follows. Section 2 is devoted to the introduction of the fast fractional differencing algorithm and the description of the simulation experiment. Section 3 presents and discusses the results of the experiment. Section 4 concludes this article and gives an overview about possible applications.

2. Methodology

Focusing on modeling the conditional variance, we set

$$\varepsilon_t = \sigma_t z_t,$$
$$\mathbb{E}\left(\varepsilon_t^2 | \mathcal{F}_{t-1}\right) = \sigma_t^2,$$

with $z_t \sim \mathcal{N}(0,1)$ i.i.d. for all $t = 1, \ldots, n$, as general framework for the following analysis.[1] In what follows, $\{\varepsilon_t\}_{t=1}^n$ is referred to as *errors* or *observations* and $\{\sigma_t^2\}_{t=1}^n$ denotes the *conditional variance* series with the sigma algebra $\mathcal{F}_{t-1}$ containing all information up to time $t-1$.

2.1. Fractionally integrated GARCH

In general, the so-called fractional difference $(1-L)^d$ of a time series $\{x_t\}_{t=1}^n$ can be written as a binomial expansion (Hosking, 1981)

$$(1-L)^d x_t = \sum_{i=1}^{\infty} \pi_i(d) x_{t-i}, \tag{1}$$

where L is the lag operator with the fractional difference parameter d and

$$\pi_i(d) = \frac{-d(1-d)\cdots(i-1-d)}{i!} \tag{2}$$

for all $i = 1, \ldots, \infty$ with $0! := 1$. This can be applied in a straightforward way to ARFIMA models[2] (Baillie, 1996).

In conditional volatility models with long memory, such as FIGARCH, the fractional differencing operator is only part of the lag structure applied on the residuals (Baillie et al., 1996). Hence, the linear convolution differs from ARFIMA models. Since the differencing is applied on ε_t^2 (and not h_t which would be the ARFIMA analogy), Eqs. (1) and (2) cannot be translated to variance models directly. A simple model setup is FIGARCH(1,d, 1) in the representation by Bollerslev and Mikkelsen (1996):

$$\begin{aligned}
\sigma_t^2 &= \omega + \left(1 - \beta L - (1-\phi L)(1-L)^d\right)\varepsilon_t^2 + \beta\sigma_{t-1}^2 \\
&= \frac{\omega}{1-\beta} + \left(1 - \frac{(1-\phi L)(1-L)^d}{1-\beta L}\right)\varepsilon_t^2 \\
&= \frac{\omega}{1-\beta} + \sum_{i=1}^{\infty} \lambda_i \varepsilon_{t-i}^2,
\end{aligned} \tag{3}$$

where

$$\begin{aligned}
\lambda_1 &= \phi - \beta - d, \\
\lambda_i &= \beta\lambda_{i-1} + \left(\frac{i-1-d}{i} - \phi\right)\left(\frac{(i-2-d)!}{i!(1-d)!}\right).
\end{aligned} \tag{4}$$

The sufficient non-negativity constraints $\omega > 0$, $0 \le \beta \le \phi + d$, and $0 \le d \le 1 - 2\phi$ have to hold in order to refer to admissible parameters. A wider range of necessary and sufficient conditions can be found in Conrad and Haag (2006). However, the discussion on conditions for weak and strict stationarity of FIGARCH is still ongoing (see further: Davidson and Li, 2014). In order to obtain non-exploding processes, we impose the condition $\sum_{i=1}^{\infty} \lambda_i < 1$.

Calculating the fractional difference of the time series with weights given in Eq. (4), a linear convolution of the residual series and the corresponding weights λ_i is performed. Naturally, the summation has to be terminated at a fixed index. This time is referred to as *truncation lag* and is denoted by n_{trunc}. In literature, chosen truncation lags range from 100 to

[1] The underlying Normal distribution for the random variable z_t is chosen for simulation purposes only and could vary in an application to account for stylized facts, e.g. heavy tails.

[2] In technical applications, this class of models is sometimes referred to as FARIMA models, see Burnecki (2012).