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Closed-form solutions for options with random initiation under asset price monitoring

Doobae Jun[a,*], Hyejin Ku[b]

[a] Department of Mathematics and Research Institute of Natural Science, Gyeongsang National University, Jinju, 660–701, Korea
[b] Department of Mathematics and Statistics, York University, Toronto, ON, Canada

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ABSTRACT

This paper studies derivatives to prepare for financial risk from unexpected events. It is difficult for firms and financial institutions to hedge losses triggered by natural catastrophes such as earthquakes, by using derivative securities with fixed initiation and maturities. In this context, we consider an option that is initiated at random by an unexpected event, and moreover, is connected with a barrier of knock-in or knock-out type for asset price monitoring until the time of event. We derive closed-form valuation formulas for these options.

1. Introduction

Financial institutions have been increasingly making use of barrier options, digital options and other path-dependent options. These options are used for a wide range of hedging, risk management and investment purposes as explained in Goldman et al. (1979) and Hull (2006). Furthermore, in recent years many insurance companies have developed more innovative saving products, and these products usually have features of barrier option. In other words, they allow the policyholder to participate in favourable investment performance while maintaining a floor guarantee on the benefit level.

Barrier options have become increasingly popular because they are more flexible and cheaper than vanilla options. Also, they have been created to provide the insurance value of an option without charging as much premium. Many papers provided pricing formulas for various types of barrier options since Merton (1973) (See for example Rich (1997) and Pelsser (2000)). For more complicated barrier options, partial barrier options which is monitored for a part of the option's lifetime are studied in Heynen and Kat (1994) and Hui (1997), where the ending time of monitoring is different from the expiry date of the option. But, the expiry date as well as the ending time of monitoring are predetermined dates.

This paper investigates derivatives which can play a role as an insurance for unexpected events such as climate extremes or earthquakes. Normally, firms and financial institutions hedge risks for their portfolios using derivatives with fixed maturities. If they wish to hedge the financial risk posed by natural catastrophes, they can not utilize contracts such as digital options with fixed starting time and maturity as in the literature on derivatives, because it is uncertain when a catastrophe

* Corresponding author. Fax: +82 55 772 1429.
E-mail addresses: dbjun@gnu.ac.kr (D. Jun), hku@mathstat.yorku.ca (H. Ku).

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occurs. Jarrow (2010) provided a closed form solution for valuing Cat bonds to manage the risks of catastrophe event losses. Jaimungal and Wang (2006) derived a pricing formula for European catastrophe equity put options under the assumption that the volatility of the stock price is constant and that the amounts of catastrophe losses have a general distribution. Jiang et al. (2013) presented a novel catastrophe option pricing model that considers counterparty risk. In this paper, we consider financial contracts that initiate immediately at a time when a catastrophe occurs, and that have the feature of asset price monitoring with a knock-in or knock-out barrier.

The exponential distribution has been widely used in various categories including life testing, reliability, and operations research owing to its advantages which are nice mathematical form and memoryless property. It is well known that an exponential random variable can be applied to the waiting time until the first event and the lifetime of an item that does not age. See for example Epstein and Sobel (1954) and Embrechts and Schmidli (1994). In this context, we study a derivative which initiates at an exponential random time and matures thereafter, and contains knock-in or knock-out barriers for asset monitoring in advance. Jun and Ku (2013) studied digital barrier options with an exponential random time, in which barrier option is given to an investor after a random event. On the contrary, barrier option is used in this paper for a monitoring purpose before the event.

The outline of the paper is as follows. Section 2 provides the economic rationale for the options considered in this paper. Section 3 presents pricing formulas for digital options linked with down-and-in and down-and-out barrier options, and a graph in Section 3 shows the properties of the solutions. Conclusion is provided in Section 4.

2. Economic rationale

Risk arises from natural events, such as earthquakes, floods and hurricanes. For example, in March 2011, the earthquake in northeast Japan caused a huge property loss and many firms have suffered from the natural disaster. Also, strong earthquakes causing severe damage to the California area have occurred a number of times in the past.

Suppose that a natural catastrophe such as a large earthquake may occur in a developed country, and that a firm in the country is concerned with the financial risk that can be posed by a large earthquake. Then the firm may want to make the following contract with a financial institution as part of their risk management strategy: A digital option will be provided in the event of a large earthquake, i.e., a fixed amount of money will be paid out at maturity (at a fixed time after the event), with the requirement that the underlying asset price never falls to reach a specified barrier until the time of earthquake. This condition may be an indication that the firm has not been in financial difficulties before the event. As long as the barrier is not hit, the contract is kept intact. Also, to reduce the magnitude of the premium the firm might add the condition that the underlying asset price must be under the specified level at maturity. The motivation is that the earthquake actually dealt a severe blow to the firm. Thus, the contract is very useful for the directly and substantially affected firm from the earthquake.

The firm pays a premium to obtain this digital option contract linked with knock-out barrier in preparing emergency. Since this type of contract has the features of both barrier and digital options, it costs much less, but is useful for preventing risks posed by random events like a large earthquake. The pricing formula for such an option is derived in Corollary 3.4.

3. Options with random initiation under asset price monitoring

Suppose that r is the risk-free interest rate and $\sigma > 0$ is constant. We assume the price of the asset S_t follows a geometric Brownian motion $S_t = S_0 \exp\left(\left(r - \sigma^2/2\right)t + \sigma W_t\right)$ where W_t is a standard Brownian motion under the risk-neutral probability P.

Let $X_t = \frac{1}{\sigma}\ln(S_t/S_0)$ and $\mu = \frac{r}{\sigma} - \frac{\sigma}{2}$. Then $X_t = \mu t + W_t$. We define the minimum and the maximum for X_t to be

$$m_a^b = \inf_{t\in[a,b]}(X_t) \quad \text{and} \quad M_a^b = \sup_{t\in[a,b]}(X_t)$$

and denote by E^P the expectation operator under the P-measure.

Suppose that τ is an exponential random variable with parameter λ and a barrier option with the lifetime of length τ is initiated at time 0, i.e., the monitoring period for asset price barrier is $[0,\tau]$. We define $d = \frac{1}{\sigma}\ln(D/S_0)$, $u = \frac{1}{\sigma}\ln(U/S_0)$ and $k = \frac{1}{\sigma}\ln(K/S_0)$ where $D(\le S_0)$ is a down barrier, $U(\ge S_0)$ is a up barrier and K is a strike price.

Consider a barrier option of knock-in type followed by another option which pays out the amount of A if the underlying asset price falls to reach the barrier D in $[0,\tau]$ and is greater than the strike price K at time $\tau + T$. The payoff of the option is zero if the underlying asset price does not hit the barrier D or falls below the strike K at time $\tau + T$. We derive a closed-form formula for the price of this option.

Theorem 3.1. *The value V_1 of a digital option connected with a down-and-in option which terminates at exponential random time τ with parameter λ is*

$$\begin{aligned} V_1 = {} & A\left[a_1 e^{-rT+\left(\mu+\sqrt{2(\lambda+r)+\mu^2}\right)d}N(d_1) - a_1 e^{\lambda T+\left(\mu+\sqrt{2(\lambda+r)+\mu^2}\right)k}N(d_2)\right] \\ & - A\left(\frac{D}{S_0}\right)^{\frac{2}{\sigma}\sqrt{2(\lambda+r)+\mu^2}}\left[a_2 e^{-rT+\left(\mu-\sqrt{2(\lambda+r)+\mu^2}\right)d}N(d_1) + a_2 e^{\lambda T+\left(\mu-\sqrt{2(\lambda+r)+\mu^2}\right)k}N\left(d_2 + \frac{2k-2d}{\sqrt{T}}\right)\right] \end{aligned}$$