Finance Research Letters 20 (2017) 146–152

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Bayesian testing for short term interest rate models

Yonghui Zhang[a], Zhongtian Chen[b], Yong Li[b,1,*]

[a] School of Economics, Renmin University of China, Beijing, 100872, PR China
[b] Hanqing Advanced Institute of Economics and Finance, Renmin University of China, Beijing, 100872, PR China

ARTICLE INFO

Article history:
Received 11 October 2015
Accepted 16 September 2016
Available online 19 October 2016

JEL classification:
C11
C12
G12

Keywords:
Bayes factor
χ^2 test
Markov Chain Monte Carlo (MCMC)
Short-term interest rate models
Hypothesis testing

ABSTRACT

In empirical finance, interest rate models have been widely used for modeling short-term interest rate. Under the framework of the hypothesis testing, this paper provides a Bayesian approach for comparing a range of alternative models. These compared models are nested in a general single-factor diffusion process for the short-term interest rate, with each alternative model indexed by the level effect parameter for the volatility. The performance of the developed procedure is illustrated by an empirical example of Eurodollar deposit rates.

1. Introduction

In theoretical and empirical finance, the interest rate plays a vital role in pricing the fixed-income securities and other financial products. It reveals the changes of financial market and provides useful information for financial practitioners or the center bank. Hence, it is important and fundamental to model the short-term riskless interest rate. Although many alternative short-term interest rate models have been proposed, such as, Merton (1973), Brennan and Schwartz (1980), Vasicek (1977), Dothan (1978) etc., the answers to which model is the best among these models for different financial markets or different datasets of interest rates are still important concerns for researchers.

In the literature, in response to this problem, many researchers have proposed all kinds of approaches for comparing those short-term interest rate models. One of the most famous papers is Chan et al. (1992) where the generalized method of moments approach is used to estimate the models. Chan et al. (1992) provided a simple econometric framework to compare the performance of the well-known models. Other researchers, such as Jones (2003) and Sanford and Martin (2006) did similar researches based on Eurodollar rates data and Australian BAB rates data. Especially, under Bayesian framework, Sanford and Martin (2006) used Bayes factor for comparison and pointed out that the different *dt* specified to discretize the continuous models led to different Bayesian estimation results.

---

[*] Corresponding author.
E-mail address: gibbsli@ruc.edu.cn (Y. Li).

[1] Li gratefully acknowledges the financial support by the Fundamental Research Funds for the Central Universities, and the Research Funds of Renmin University of China (Grant no. 14XNI005).

http://dx.doi.org/10.1016/j.frl.2016.09.020
1544-6123/

Table 1
Parameter restrictions of short-term interest rate models.

Model	α	β	σ^2	γ
Merton		0		0
Vasicek				0
CIR-SR				0.5
Dothan	0	0		1
GBM	0			1
Brennan–Schwartz				1
CIR-VR	0	0		1.5
CEV	0			

In this paper, we follow the idea of Sanford and Martin (2006) and focus on providing a new approach for comparing the alternative interest rate models. The inferential approach is still based on the Bayesian framework. Coupled with Markov Chain Monte Carlo (MCMC) techniques, analysis of the interest rate models is done on the basis of the Bayesian posterior outputs. Instead of using the well-known Bayes factor, we establish our empirical comparing scheme under the hypothesis testing framework. The reason is that the Bayes factor suffers from the notorious Jeffreys–Lindley's paradox which means that it always supports the null hypothesis when the prior distribution has a large variance spread. Furthermore, it can not be defined under improper prior distributions. For more details, on can refer to Kass and Raftery (1995), Li et al. (2014).

In a recent paper, Li et al. (2015) proposed a new Bayesian test statistic to replace the Bayes factor in point-null hypothesis testing. It can be shown that the new statistic at least enjoys four desirable properties. First, it is well-defined under improper distributions. Second, it is immune to the so-called Jeffreys–Lindley's paradox. Third, it is easy to compute. Fourth, the asymptotic distribution is pivotal, which always follows the χ^2 distribution. Hence, it is very convenient to be implemented in practice.

In the present paper, the lately proposed Bayesian test statistic by Li et al. (2015) is adopted for testing the parameters in short-term interest rate models to implement the empirical comparison. In these interest rate models, the closed form of the likelihood functions is available, then, the first derivatives of the observed log-likelihood function is also easy to get. In addition, the posterior means of the parameters can be easily obtained from MCMC outputs. Hence, the Bayesain test statistic is very simple to be calculated to determine whether the hypothesis is accepted or rejected. Eventually, the dataset used for illustration is the one-month Eurodollar deposit rate which is stationary and mostly satisfies the assumption of interest rate liberalization in the short-term interest rate models.

The rest of this paper is organized as follows. Section 2 introduces Bayesian analysis of interest rate models. Section 3 describes the Bayesian test statistic for hypothesis testing. Section 4 illustrates the new approach by using Eurodollar deposit rates data. Section 5 concludes the paper.

2. Bayesian analysis of interest rate models

As shown in Chan et al. (1992), there is a very generalized single-factor diffusion process model (CKLS, hereafter) which can nest a variety of the well-known short-term interest rate models. It can be described by the following stochastic differential equation:

$$dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ, \tag{1}$$

where r is the short-term riskless rate, t is the time, dZ is the independent increments of a Wiener process and α, β, γ, σ are the indexed parameters. The nested models can be indexed by the level effect parameter γ on the volatility.

In this paper, we shall focus on estimating and testing the eight widely-applied short-term interest rate models as well as the generalized model. The eight models are listed below and can be obtained from (1) by imposing appropriate parameter restrictions as shown in Table 1. As for more details about these models, one can consult to Chan et al. (1992) and its references therein.

1. Merton	$dr = \alpha dt + \sigma dZ$
2. Vasicek	$dr = (\alpha + \beta r)dt + \sigma dZ$
3. CIR-SR	$dr = (\alpha + \beta r)dt + \sigma r^{\frac{1}{2}} dZ$
4. Dothan	$dr = \sigma r dZ$
5. GBM	$dr = \beta r dt + \sigma r dZ$
6. Brennan–Schwartz	$dr = (\alpha + \beta r)dt + \sigma r dZ$
7. CIR-VR	$dr = \sigma r^{\frac{3}{2}} dZ$
8. CEV	$dr = \beta r dt + \sigma r^{\gamma} dZ$