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A Unified Tree approach for options pricing under stochastic volatility models

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ABSTRACT

We develop a simple and efficient tree approach for pricing options under stochastic volatility. Our method encompasses the models of Heston, Hull-White, Stein-Stein, α-Hypergeometric, 3/2 and 4/2 models. Numerical results are provided to illustrate the effectiveness of the proposed method.

1. Introduction

Given the numerical tractability and good properties of the tree method, since the pioneer work of Cox et al. (1979), several generalizations have been introduced. Examples are the trees of Amin (1993), Hilliard and Schwartz (2005), Derman and Kani (1998). Each generalization aims to price options consistently with the observed implied volatility smile/skew and/or term structure. In addition, all generalizations attempt to maintain the simplicity of the model by controlling dimensionality.

Options pricing in stochastic volatility (SV) models using trees have been considered in the literature. For example, Vellekoop and Nieuwenhuis (2009) develop a tree-based algorithm to price options in Heston (1993)'s model. Beliaeva and Nawalkha (2010) present a multidimensional tree to price American options in Heston's model. Recently, Liu (2010), using a partial differential equation (PDE)-Markov chain approximation to price options under Heston's model. However, the tree-based algorithm of Vellekoop and Nieuwenhuis (2009) grows quadratically while the trees of Beliaeva and Nawalkha (2010) and Liu (2010) are limited to Heston's model and it is not straightforward to extend their methods to other models such as 3/2, Hull-White. In addition to the tree method, finite scheme methods for SV models have been extensively studied in the literature as it seems to be more flexible to price options in this class of models. For example, under Heston's model,

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numerical methods based on PDE are considered in Chiarella et al. (2009),Salmi et al. (2014). For a good account of the literature of PDEs method, we refer the interested reader to Salmi et al. (2014) and references therein. In practice, we prefer intuitive and simple methods. When taking these into account, the tree method has become one of referred methods to calculate the price of European options and American options.

Since the pioneer work of Hamilton (1994), regime switching models have been prevalent in the options pricing literature due to their simplicity and analytically tractable properties. By allowing the model parameters, such as the volatility, rate of return, interest rate, and dividend rate, to take different values in different regimes, regime switching models can capture more accurately the change in macro-market conditions, while, at the same time, preserve to a certain degree the simplicity of the model. The existing literature on pricing options using trees is quite rick. Examples include those of Yuen and Yang (2010), Costabile et al. (2014), Jiang et al. (2016) and references therein.

Options pricing in SV models is notoriously challenging due to the correlation structure of the two Brownian motions. In this paper, we show that we first can remove the correlation between two Brownian drivers; the resulting system then can be weakly approximated by a regime switching model using a continous time Markov chain approximation[1]. Under the new regime switching model, both European and American exercise style options can be priced using a tree method. The blend of a regime-switching model and Markov chain approximation (MCA) techniques results in a recombined tree whose the nodes only grow linearly. As a result, the proposed lattice method provides a unified approach for a wide class of SV models, including those of Heston, Hull-White, Stein-Stein, α-Hypergeometric, 3/2 and 4/2 stochastic model. We note that our approach is different from that of Leccadito et al. (2012), and of Liu and Nguyen (2015) as the authors do not explain whether their tree approach can be extended to handle non-affine models like 3/2 or 4/2 model, Hull-White's model, or Stein-Stein's model. Moreover, Liu and Nguyen (2015) adopt the approach of Hilliard and Schwartz (2005), which leads to solving a linear system whose jump probabilities are not guaranteed to be in [0, 1]. This is a major drawback of their method. In this paper, we show the good features of the proposed methodology in a option valuation study under both European and American exercise features.

The rest of the paper is structured as follows: In Section 2 we motivate the tree method using the 3/2 stochastic volatility model. Section 3 is concerned with options pricing using a trinomial tree. In Section 4, we provide the dynamics of several stochastic volatility models considered in this paper. Numerical examples to illustrate the effectiveness of the tree method are reported in Section 5. The paper is concluded with some remarks in Section 6.

2. Stochastic volatility

2.1. A motivating example: 3/2 stochastic volatility model

Consider the 3/2 stochastic volatility model whose dynamics is given by

$$\begin{cases} dS_t = S_t r dt + S_t\sqrt{v_t}dW_t^1, \\ dv_t = v_t[\eta(\theta - v_t)dt + \sigma_v\sqrt{v_t}dW_t^2], \end{cases} \tag{1}$$

where in (1) v_t is the variance of the asset S_t, r is the risk-free interest rate, $\sigma_v > 0$, $\theta \in \mathbb{R}$ is the mean reversion level, η is given such that ηv_t-a stochastic volatility quantity- is the speed of mean reversion, W_t^1 and W_t^2 are two one-dimensional standard Brownian motions satisfying $E[dW_t^1 dW_t^2] = \rho dt$; where $\rho \in (-1, 1)$ is the correlation coefficient between the two Brownian motions.

Clearly, it is not easy to find the option value by using the dynamics of S_t directly. Our goal is to overcome this difficulty by converting the 3/2 model to a regime switching diffusion model. From there, both European and American options can be priced. To this end, use Ito's lemma we have

$$d\left(\frac{1}{v_t}\right) = \eta\theta\left(\frac{\eta + \sigma_v^2}{\eta\theta} - \frac{1}{v_t}\right)dt - \frac{\sigma_v}{\sqrt{v_t}}dW_t^2. \tag{2}$$

Hence if we let $\widehat{v}_t = \frac{1}{v_t}, \widehat{\eta} = \eta\theta, \widehat{\theta} = \frac{\eta+\sigma_v^2}{\eta\theta}, \widehat{\sigma_v} = -\sigma_v$, then (1) becomes

$$\begin{cases} dS_t = S_t r dt + \frac{S_t}{\sqrt{\widehat{v}_t}}dW_t^1, \\ d\widehat{v}_t = \widehat{\eta}(\widehat{\theta} - \widehat{v}_t)dt + \widehat{\sigma}_v\sqrt{\widehat{v}_t}dW_t^2, \ \widehat{v}_0 = 1/v_0. \end{cases} \tag{3}$$

Define

$$X_t = \log\left(\frac{S_t}{S_0}\right) - \frac{\rho}{\widehat{\sigma}_v}\log\left(\frac{\widehat{v}_t}{\widehat{v}_0}\right) - rt, \tag{4}$$

[1] In Lo and Skindilias (2014) and Lo et al. (2016) the authors show the good approximation features of a Markov chain approximation for processes with known or unknown density funtcions. See Simonato (2011) for more applications in options pricing.