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Discontinuous payoff option pricing by Mellin transform: A probabilistic approach

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ABSTRACT

The Mellin transform technique is applied for solving the Black-Scholes equation with time-dependent parameters and discontinuous payoff. We show that the option pricing is equivalent to recovering a probability density function on the positive real axis based on its moments, which are integer or fractional Mellin transform values. Then the Mellin transform can be effectively inverted from a collection of appropriately chosen fractional (i.e. non-integer) moments by means of the Maximum Entropy (MaxEnt) method. An accurate option pricing is guaranteed by previous theoretical results about MaxEnt distributions constrained by fractional moments. We prove that typical drawbacks of other numerical techniques, such as Finite Difference schemes, are bypassed exploiting the Mellin transform properties. An example involving discretely monitored barrier options is illustrated and the accuracy, efficiency and time consuming are discussed.

1. Introduction

Barrier features in option contracts give a lot of opportunities to investors. It is well-known that the option valuation problem is more complicated in case of discontinuous payoffs together with application of discrete barriers. Some classical quantitative methods, for example the Crank-Nicolson finite difference scheme, suffer from drawbacks such as negative option prices due to undesired spurious oscillations in the numerical solution (Milev and Tagliani, 2013). This problem could be solved by applying a significantly small time step and that makes the Crank-Nicolson method time-consuming. Similarly to the finite difference scheme approach, the finite element method is more sophisticated and efficient for discontinuous payoff option pricing (Ballestra and Pacelli, 2011). D. Jun proposes an accurate continuity correction approximation for options with two barriers applied at a finite set of times (Jun, 2013) while Fang and Oosterlee suggest a pricing method based on Fourier-cosine expansions for early-exercise and discretely-monitored barrier options (Fang and Oosterlee, 2009). The method works well for exponential Levy asset price models. Efficient recombining quadrature methods for discrete barriers are compared in (Hong et al., 2015; Milev and Tagliani, 2010b).

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Farnoosh et al. (2015) explore this problem under the BlackScholes framework but with time dependent parameters, e.g. volatility, interest rate and dividends. Such option contracts have been efficiently priced with Gauss, Laplace, Mellin and other transform methods (Yoon and Kim, 2015; Kim et al., 2015; Feng and Linetsky, 2008) over the past decade. Options under a stochastic (HullWhite) interest rates are efficiently priced by a double Mellin transform (Yoon and Kim, 2015). In this paper we investigate the use of Mellin transform to non-standard option pricing models, characterized by discontinuities in the terminal/boundary conditions and with time-varying parameters. Allowing the parameters (risk-free interest rate, volatility and dividend yield) to vary with time leads to a more flexible model that could capture the market's view concerning the direction of the variables future behavior upon which the option value depends. Despite the immense interest for using transforms to compute the value of options, the Mellin transform has received little attention. This may partly due to the fact that the PDE for pricing is often formulated in terms of log-prices. On the contrary, the Mellin transform enables option equations to be solved directly in terms of market prices rather than log-prices, providing a more natural setting to the problem of pricing. The Mellin transform's ascension into the realm of Financial mathematics is only about two decades old, see for example Cruz-Báez and Gonźalez-Rodríguez (2002) just to mention the pioneering one. A few papers, considering several kinds of options driven by different stochastic processes, have been published recently (Xiao and Ma, 2016).

We propose an alternative Mellin transform inversion technique in this paper. By a proper scaling, the Black-Scholes equation positive solution can be considered a probability density function (pdf) with known moment curve (the Mellin transform). Then, the option pricing is led to recovering a pdf knowing its integer (or non-integer) moments. The MaxEnt machinery may be invoked for choosing the approximating density.

Below we concentrate our attention on the use of Mellin transform for pricing barrier options with a discrete monitoring clause. They are characterized by discontinuous payoff and discontinuities that are renewed at each monitoring date t_i. In general, finding an expression for the solution of Black-Scholes equation when the coefficients or payoff function are piecewise continuous ordinary functions or generalized function is not an easy matter.

The proposed method, i.e. Mellin transform inversion by means of MaxEnt technique, results in

1. an option pricing approximate solution that is positivity preserving according to the financial meaning of the underlying problem;
2. an accurate option pricing based on the previously known MaxEnt technique underlying results of convergence.
3. competitiveness whenever few monitoring dates as well as high values of maturity T, are considered. Indeed, the method is used at each monitoring date t_i and (as usual in the Integral Transforms setup) it is independent on the time span $t_{i+1} - t_i$ between successive monitoring dates, equivalently there is no need to consider intermediate time steps.
4. the possibility of observing the life of the option for each monitoring date providing an approximate solution through an analytical expression. As a consequence, the computation of Greeks, such as Delta $\frac{\partial V}{\partial S}$ and Gamma $\frac{\partial^2 V}{\partial S^2}$, is easily obtained.
5. The numerical diffusion, arising from low volatility and affecting Finite Difference and Finite Element technique is closely related to discontinuous payoff, as with the placement of the nodes in the lattice and barrier location affecting the tree methods, are absent too. Summarizing, spurious oscillations, numerical diffusion, placement of the nodes in the lattice and barrier location, are drawbacks which do not affect the numerical solution obtained by Mellin transform technique.

The structure of the paper is as follows. In Section 2 the model is presented. In Section 3 the numerical solution of Black-Scholes equation, the Mellin Transform and MaxEnt technique are illustrated. In Section 4 some numerical results are presented. In Section 5 we give conclusions.

2. The model

In order to make our analysis concrete we have concentrate our attention on a barrier option with a discrete monitoring clause, but the analysis presented can be pairwise extended to many other exotic contracts (digital, supershare, binary and truncated payoff options, callable bonds and so on). For example, for a double barrier knock-out call option, the payoff condition is continuous and equal to $(S-K)^+$ but the option expires worthless if before maturity the asset price falls outside the corridor $[L, U]$ at the predefined monitoring dates. The Black Scholes equation is applied in the intermediate periods over the real positive domain. In our model, we consider the case of a dividend paying stock with a price S, which evolves according to geometric Brownian motion $dS = [\mu(t) - D(t)]S(t)dt + \sigma(t)S(t)dW_t$, where the drift rate $\mu(t)$, volatility $\sigma(t)$ and dividend yield $D(t)$ are assumed to be time-dependent and with W_t being the standard Brownian motion. The contract to be priced is a discretely monitored knock-out double barrier call option. If t is the time to expiry T of the contract, the price $V(S, t)$, $0 \le t \le T$, of a derivative claim on S satisfies the Black-Scholes PDE

$$-\frac{\partial V}{\partial t} + [r(t) - D(t)]S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2(t)S^2\frac{\partial^2 V}{\partial S^2} - r(t)V = 0 \tag{2.1}$$

in each interval (t_{i-1}, t_i), where the $t_1 < \ldots < t_F = T$ are the times to default at each monitoring. Here $r(t)$ and $\sigma(t)$ are strictly positive, piecewise continuous functions on $[0, T]$, and $D(t)$ is non-negative function on $[0, T]$. Eq. (2.1) is endowed by its initial and boundary conditions

$$V(S,0) = (S-K)^+\mathbf{1}_{[L,U]}(S), \qquad V(S,t) \to 0 \text{ as } S \to 0 \text{ or } S \to \infty. \tag{2.2}$$