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Value-at-Risk estimation with stochastic interest rate models for option-bond portfolios

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ABSTRACT

This article proposes a Monte Carlo simulation based approach for measuring Value-at-Risk of a portfolio consisting of options and bonds. The approach allows for jump-diffusions in underlying assets and affords to fit a variety of model layout, including both non-parametric and semi-parametric structures. Backtesting was conducted to assess the effectiveness of the method. The algorithm was tested against various trading positions, time horizons, and correlations between asset prices and market return rates. A prominent advantage of our approach is that its implementation does not require prior knowledge of the joint distribution or other statistical features of the related risk factors.

1. Introduction

McNeil et al. (2005) defined the Value-at-Risk (VaR) of a portfolio as the smallest number l that the probability of the loss L of the portfolio exceeds l is not larger than $1-\alpha$ at a given confidence level $\alpha \in (0, 1)$. Mathematically, it is denoted as $\mathrm{VaR}_\alpha = \inf\{l \in \mathbb{R} : P(L > l) \le 1-\alpha\} = \inf\{l \in \mathbb{R} : F_\alpha \le \alpha\}$. In another word, VaR places an upper bound of the potential loss of an interested portfolio at a given confidence level, where a higher confidence level indicates a smaller probability that the loss may be exceeded. Time series models are well-known tools to estimate VaR. For instance, Longin (2000); Danielesson and De Vries (2000) estimated unconditional VaR by combining non-parametric historical simulation with parametric estimation based on extreme value theory. Their VaR approaches heavily relied on the prediction of extreme events. However, Embrechts (2000) demonstrated that it is a challenging task to predict the extreme events. GARCH is one of most commonly used time series models (Fan and Gu, 2003; Wilhelmsson, 2009; Jánský and Rippel, 2011). In particular, Gao and Song (2008) estimated VaR by the asymptotic behavior of residual distribution functions with GARCH models. Though the results are analytical, the asymptotic distributions become intractable when an arbitrarily long horizon is concerned. Moreover, Berkowitz and Brien (2002) found that ARMA-GARCH model did not provide satisfactory forecast of VaR when it

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was applied to predict banks' daily profit and loss (P & L). By extending the linear time series to nonlinear time series Chiu and Chuang (2016) examined the effectiveness of the Switching forecast model in estimation of VaR in Asian stock market. The major drawback of the Switching forecast model is its assumption of the existence of quadratic loss function, which may require theoretical and/or empirical justifications for a more generalized application.

To replicate the price of the corresponding financial derivatives is another widely used approach to simulate VaR, the validity and efficiency of which are largely affected by the fitness of the proposed models. In line of this direction, Abudy and Izhakian (2011) priced stock options with a stochastic interest rate process. Although their method simplified the valuation procedure, it is not applicable to estimate portfolio VaR within a t-day horizon due to information loss. Burgess (2014) used Vasicek model to price a bond option, defined as an option to buy or sell a bond at certain prescribed price, constituting a special case of the problem addressed by the current paper. Fang (2012) discussed European option pricing under Vasicek model by partial differential equations; but this method is not equally tractable for other mean-reverting models. Kim and Kunitomo (1999) and Kim (2002) proposed the small-disturbance asymptotics for the Itô process to price options. They are not fully adequate to estimate VaR because of the absence of time filtration.

In our work, loss function will not be constructed, as it is unnecessary. VaR will be directly estimated with Monte Carlo simulation algorithms based on the analytical results derived for both Vasicek and CIR models under the Black–Scholes framework, giving rise to a major advantage over time series techniques. Calibrated with real market data, the paper estimates VaRs of the two representative portfolios in representative horizons along with the sensitivity analysis, and compares the estimated results with those simulated from historical data. Section 2 provides the analytical results for the price of the options making up the assumed portfolios in this study, where the underlying stock is assumed to follow the Vasicek or the CIR interest model. In the same section, a brief description is given regarding the testing data, the methodology, and the main algorithms. Section 3 provides numerical examples and outputs of our algorithms using representative portfolios. The advantages of our approaches are empirically demonstrated through comparative analysis with other methods. Section 4 summarizes the current work, where a possible future direction is proposed.

2. VaR model structure

2.1. Vasicek model

By assuming that the interest rate r_t follows the Vasicek model and stock price S_t follows the geometric Brownian motion, we construct our model as

$$\begin{aligned} dr_t &= \alpha(\theta - r_t)dt + \sigma_r dW_t \\ dS_t &= S_t(r_t dt + \sigma_s dZ_t) \end{aligned} \tag{1}$$

where $dW_t = dW_t^*,\ dZ_t = \rho dW_t^* + \sqrt{1-\rho^2}dZ_t^*,\ W_t^*$ and Z_t^* are two independent Brownian motions, and ρ is the correlation between Brownian motions W_t and Z_t.

2.1.1. Zero coupon bond pricing

The arbitrage-free price of a zero coupon bond, when the interest rate is assumed to follow the above Vasicek model, is given by $P(t,T) = e^{-A(t,T)-B(t,T)r_t}$ (Filipović, 2009), where

$$\begin{aligned} B(t,T) &= -\frac{1}{\alpha}\left(e^{-\alpha(T-t)} - 1\right) \\ A(t,T) &= \sigma_r^2\frac{4e^{-\alpha(T-t)} - e^{-2\alpha(T-t)} + 2\alpha(T-t) - 3}{-4\alpha^3} + \theta\frac{e^{-\alpha(T-t)} - 1 + \alpha(T-t)}{\alpha} \end{aligned} \tag{2}$$

Because the market price of risk, commonly denoted as Θ, in the Vasicek model is assumed to be constant on a finite time horizon, the discounted price of the underlying assets remains a martingale. According to Cheridito et al. (2005), this accredits to price the zero coupon bond using the real measure with the same structure.

2.1.2. European option price

The analytical form of the price of the European call option is derived as follows. First, the HJB framework condition is written as

$$\alpha^*(t,T) + \frac{1}{2}|\sigma^*(t,T)|^2 - \sigma^*(t,T)\theta_t = 0 \tag{3}$$

where $\alpha^*(t,T) = -\int_t^T \alpha(t,u)du$ and $\sigma^*(t,T) = -\int_t^T \sigma(t,u)du$. Under this condition, the dynamic forward curve has the form

$$df(t,T) = -\sigma(t,T)|\sigma^*(t,T)|^\top dt + \sigma(t,T)dW_t^Q \tag{4}$$