



# How fundamental is the one-period trinomial model to European option pricing bounds. A new methodological approach<sup>☆</sup>



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## ABSTRACT

We offer a new simple approach to price European options in incomplete markets using the sole no-arbitrage principle and this only requires to make use of a one-period model; introducing a stochastic process is unnecessary. We show that determining the range of arbitrage-free prices with a trinomial model only consists in locating two points on a triangle. As this range of prices may be lower than the classical ones, the parameters of the model can be implied from the quoted bid and ask prices of liquid European options, used in turn to estimate the volatility bounds. A simple example is provided using options on the S & P 500.

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## 1. Introduction

A European option (call, put...) written on a financial asset such as a stock or a stock index is a contract that gives its holder at maturity the right but not the obligation to buy or to sell the stock (or the index) at a pre-determined price called the strike price. From the seminal paper of Merton (1973), it is common to value options (and more generally derivatives) using the so-called no-arbitrage principle, which basically states that an investment strategy that can be implemented at no cost should not generate positive gains only (see e.g., Varian, 1987; Staum, 2007). When one considers a specific probabilistic model (popular ones are binomial or Black-Scholes), the no-arbitrage alone leads to a unique price because the option payoff can be perfectly replicated using the basic traded assets. Markets are said to be complete. Perfect replication is however not possible in general and the application of the no-arbitrage principle leads to a *range of prices* rather than to a unique price. Markets are said to be incomplete.

In practice, the causes of market incompleteness are related to trading constraints, for instance the impossibility (or the difficulty) to sell short a security (see Staum, 2007; Sonono and Mashele, 2016). In theory, the causes of market incompleteness are generally not explained; they are simply related to the characteristics of the underlying stochastic process.

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For instance, in the standard environment composed of a risky stock and a default risk-free asset, markets are (in general) incomplete when the stock price is assumed to follow a (discrete-time) trinomial process (see e.g., [Van der Hoek and Elliott, 2006](#), Appendix B) or a (continuous-time) Levy process (see e.g., [Cont and Tankov, 2004](#), chapter 10).

In the literature, there are various pricing/hedging topics (e.g., option pricing bounds, indifference pricing, quadratic hedging...) that are analyzed under a different set of assumptions but their common point is that a particular stochastic process for the risky asset is given. For instance, in their seminal theory paper, [Eberlein and Jacod \(1997\)](#) consider a Levy process and exhibit the set of arbitrage-free prices for European options with convex payoffs. In their numerical methods oriented paper, [Xiao and Ma \(2016\)](#) also consider a Levy process but to price a double barrier option. [Brown et al. \(2015\)](#) consider the determination of the pricing bounds of path-dependent European options using a trinomial process while [Fard and Siu \(2013\)](#) consider the valuation of European options using a Markovian regime switching binomial process. [Sundaram and Das \(2015, chapter 16\)](#) presents a lucid and very readable overview of the various types of (discrete time/continuous-time) processes that includes stochastic volatility models (see also [Shi et al., 2016](#) where a non-affine stochastic volatility model is considered).

In this paper, we want to dispute the need to introduce a stochastic process (and even a probabilistic model) for standard European options valuation. Building on the methodology introduced recently in [Braouezec and Grunspan \(2016\)](#) but restricted to standard European options (i.e., non path-dependent), the determination of the option pricing bounds only requires to specify the set of possible stock (and option) prices at maturity  $T$ ; intermediate values are irrelevant. We focus here on the simplest set that contains three possible prices (hence the name of trinomial model) and we show that the determination of the option pricing bounds simply consists in locating two points on a triangle. The interesting point is that the resulting pricing bounds may be much lower than the classical ones so that they may coincide with observed bid and ask prices. For instance, ([Eberlein and Jacod, 1997](#)) note that the no arbitrage principle alone, when the stock price is assumed to follow a Levy process, does not suffice to value contingent claims because the range of (arbitrage-free) prices is much larger than the interval corresponding to the bid-ask spread. We shall actually show that this property is due to the implicit strong assumption that the set of possible stock prices at time  $T$  is the *set of real numbers*, i.e., the lower bound is equal to zero and there is no upper bound. As we work with a one-period trinomial model, such a problem does not exist, and the bounds of the stock price can indeed be calibrated to the observed option prices (bid/ask). A simple example along this line is provided using options written on the S&P 500. Once this is done, one may use this information to estimate the implied volatility bounds for a given option contract.

The remainder of this paper is organized as follows. The second section is devoted to the assumptions and discussion. The third and fourth section are devoted respectively to the option pricing bounds and to a simple example that shows how to imply the volatility from the quoted option prices.

## 2. Assumptions

We consider a standard financial market in which there are two basic traded assets, a default risk-free asset, whose (deterministic) interest rate is equal to  $r > 0$ , and a risky stock. Only two dates will be considered, the current time  $t = 0$  and the  $T > 0$ . Let  $S_T$  be the stock price at time  $T$ .

**Assumption 1.** The stock price at time  $T > 0$  takes three different values

$$S_T(\omega) = \omega S_0 \quad \omega \in \Omega := \{d, m, u\} \quad (1)$$

and each state of the world  $\omega$  has a strictly positive probability.

The above assumption thus means that the underlying probability measure  $\mathbb{P}$  is such that  $\mathbb{P}(\{\omega\}) := p_\omega > 0$  for each  $\omega \in \{d, m, u\}$ . In what follows, we shall assume that  $d, m$  and  $u$  are three positive numbers such that  $d < m < u$ . Since the probability measure  $\mathbb{P}$  is not fully specified (or not fully known), it is usual to talk about *Knightian uncertainty*. Under such a situation, it is thus not possible to compute the volatility of the stock. Let  $g(S_T)$  be the payoff of the European option with maturity  $T$  and consider the three following points.

$$\mathbf{D} = (dS_0; g(dS_0)), \quad \mathbf{M} = (mS_0; g(mS_0)), \quad \mathbf{U} = (uS_0; g(uS_0)) \quad (2)$$

A given point simply represents the stock price and the option's payoff at time  $T > 0$  in given state of the world.

**Assumption 2.** The polygon spanned by the three points  $\mathbf{D}$ ,  $\mathbf{M}$  and  $\mathbf{U}$  forms a *triangle*

The polygon is either a triangle or a segment. From a financial point of view, it is only when this polygon is a triangle that markets are incomplete, and this explains the above assumption. It is interesting at this stage to understand what happens when one adds new states of the worlds. Consider the general case in which  $\Omega = \{\omega_1, \dots, \omega_n\}$  (with  $\omega_i < \omega_{i+1}$  for each  $i = 1, \dots, n-1$ ,  $n > 3$ ) so that we now have  $n$  points;  $\mathbf{P}_i = (\omega_i S_0, g(\omega_i S_0))$ ,  $i = 1, \dots, n$ . The polygon spanned by these  $n$  points is the *smallest convex polygon* that contains these  $n$  points.<sup>1</sup> When the European option  $g$  under consideration is a

<sup>1</sup> It is called more technically the convex hull, see [Braouezec and Grunspan \(2016\)](#). For an arbitrary set of  $n$  points of  $\mathbb{R}^2$  (or  $\mathbb{R}^d$  with  $d > 2$ ), the computational complexity (of the convex hull) increases with  $n$  and depends on the algorithm which is used. However, in our financial setting, the  $n$

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