Finance Research Letters 21 (2017) 115–125

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Real and complex wavelets in asset classification: An application to the US stock market[☆]

Joanna Bruzda

Nicolaus Copernicus University, Faculty of Economic Sciences and Management, Gagarina 13A, 87-100 Toruń, Poland

ARTICLE INFO

Article history:
Received 1 August 2016
Revised 7 January 2017
Accepted 1 February 2017
Available online 4 February 2017

JEL codes:
C14
C38
G11

Keywords:
Wavelet transform
Hilbert transform
Defensive asset
Cyclical asset
Financial cycle
Stock grouping

ABSTRACT

In the paper we suggest the use of wavelets to classify equities and industries into defensive and cyclical categories. We demonstrate that real- and complex-valued wavelets better serve the purpose of equity classification than more traditional approaches, and that this takes place through a more reliable and detailed dependence measurement and risk assessment. In particular, we introduce a family of wavelet-based tests of the random walk hypothesis exploring local features of spectra, which enable examining mean reversion and cyclicality of prices. The suggested approach is illustrated with an analysis of daily and monthly US industry indexes.

1. Introduction

Wavelet analysis is a recognized statistical tool with a broad range of applications, including those in social sciences. In finance this nonparametric methodology was used in examining comovements in financial markets, various forms of risk analysis, and construction of investment strategies (see, among others, Gençay et al., 2002, 2005; Fernandez, 2005, 2008; In and Kim, 2006; Conlon et al., 2008; Conlon and Cotter, 2012). These applications, however, concentrated almost exclusively on the real-valued wavelet transform and the so-called wavelet analysis of variance and covariance based on it, one among few exceptions being the study of Rua and Nunes (2009), who apply wavelet coherence analysis based on the continuous wavelet methodology to examine comovements between returns on different stock markets. In our paper, we point to further benefits of using complex-valued wavelets in financial applications, resulting from utilizing such concepts as wavelet gain, wavelet phase-locking, or wavelet amplitude correlation, and also deepen the discussion on the place of the univariate wavelet transform in risk analysis through the introduction of a family of wavelet-based tests of the random walk (RW) hypothesis enabling, among others, examination of the cyclical variability of prices or their long-term mean reversion. These concepts are applied to the task of industry classification into defensive and non-defensive groups.

☆ *This paper has greatly benefited from comments and feedback received from participants of the 2nd International Workshop on "Financial Markets and Nonlinear Dynamics" (4–5 June 2015, Paris) as well as Anna Zamojska (University of Gdańsk), and Piotr Fiszeder and Adam Tusiński (Nicolaus Copernicus University in Toruń).*

E-mail address: Joanna.Bruzda@uni.torun.pl

http://dx.doi.org/10.1016/j.frl.2017.02.004
1544-6123/

Classifications of assets and industries according to their dependence on the business cycle and the state of the market is often based on some a priori grounds or chosen financial indicators (see the classic papers by Dirks, 1958; Farrell, 1974; Martin and Klemkosky, 1976). However, as pointed out by many researchers (e.g., Vermorken et al., 2010; Costa and de Angelis, 2011), such approach does not guarantee that the groups obtained are homogenous according to their risk-return profiles. Hence, it may not always be optimal for the purpose of portfolio management. Furthermore, basing stock grouping mainly on statistical grounds should minimize the influence of subjective judgment and ex ante assumptions.

The traditional non-judgmental approach usually starts with examining beta coefficients and also covers variance comparison and a formal statistical assessment of the dependence on the business cycle.[1] However, as is well known, these methods generally lead to different risk assessments either because they measure the exposure to somewhat different risk factors (as is the case with beta coefficients and correlations with the business cycle) or assume a different coverage of these factors. The latter can be seen through the excess return market model decomposing the risk of an asset into the systematic part and the specific (idiosyncratic) risk, encompassing the influence of such risk factors as the life cycle of an industry or its dominant business model. This decomposition points out that high-beta (aggressive) stocks tend to be riskier than others, although it is obvious that the notions of risky, cyclical, and aggressive assets do not entirely coincide.

In this paper we would like to contribute to the statistical apparatus of asset classification as defensive or non-defensive under the different definitions of defensiveness. The main motivation for using wavelet analysis in risk assessment has been highlighted by Gençay et al. (2002, 2005), and Fernandez (2005, 2008), who draw attention to the benefits of employing the wavelet methodology to examine scale-dependent systematic risk. Our motivation for using wavelets in risk analysis goes somewhat further by also utilizing the excellent temporal localization and analyticity features of wavelets. This leads to a better assessment of the sensitivity to market risk at business cycle frequencies with gain coefficients correcting for lead-lag effects and to a separate examination of amplitude and phase relationships with amplitude correlation coefficients (ACCs) and phase-locking values (PLVs), computed on the base of instantaneous amplitudes and phase differences.[2] The ACCs are suggested here to examine comovements in cyclical variability, while the PLVs make it possible to find dependencies in the presence of non-synchronized amplitude changes and should be of use in constructing investment strategies based on the business cycle.[3]

Besides the use of the above mentioned coefficients of bivariate wavelet analysis, one of the ingredients of the procedure we suggest here are our wavelet-based variance difference (VD) tests of the RW hypothesis. These tests make it possible to concentrate on specific local features of univariate spectra through their targeting on rejections of the RW process due to a large or scale-dependent cyclical variance, or long-term negative autocorrelation in returns, but also calendar effects and, first of all, short-term positive return autocorrelation.

Further in the paper, we first introduce the VD tests and provide results of a small-scale simulation analysis examining their size properties for data lengths equal to those in the empirical part. Then, our empirical results for the US stock market are presented, encompassing the examination of uni- and bivariate wavelet statistics for both daily and monthly data.

2. Wavelet term profiles of risk

Let x_t be an AR(F)IMA process and $\tilde{W}_{j,t}$ and $\tilde{V}_{j,t}$ its zero-mean stationary level j wavelet and scaling coefficients obtained through the maximum overlap discrete wavelet transform (MODWT) – see Percival and Walden (2000) (hereafter [PW]), Ch. V. The variance of coefficients $\tilde{W}_{j,t}$, called the wavelet variance at scale $\lambda_j = \frac{1}{2^{j-1}}$, explains variation of oscillations with periods approximately in the interval $[2^j,\ 2^{j+1}]$. A fundamental result for the wavelet variance states that, for covariance stationary processes, it holds:

$$\mathrm{Var}(x_t) = \sum_{j=1}^{\infty} \mathrm{Var}(\tilde{W}_{j,t}) = \sum_{j=1}^{\infty} \sigma^2(\lambda_j) \tag{1}$$

(see [PW], Ch. VIII). The decomposition (1) produces the so-called wavelet spectrum – a summary of spectral properties of the examined process, leading to a piecewise constant approximation to its spectral density function $S(f)$ via the following relationship:

$$S(f) \approx 2^j \sigma^2(\lambda_j) \text{ for } f \in [\tfrac{1}{2^{j+1}},\ \tfrac{1}{2^j}].$$

[1] See *Ground Rules for the Management of the FTSE Cyclical and Defensive Index Series* (2014), Koesterich and Morillo (2013), *Sector Performance Across Business Cycles* (2009), and also compare, for example, Conover et al. (2008), Backus et al. (2010), and Yuksel and Bayrak (2012).

[2] For a detailed technical description of these concepts and some of their uses see Bruzda (2013), (2015).

[3] Compare, for example, Égert and Sutherland (2014), who show that amplitudes of real business cycles in OECD countries were becoming smaller during the Great Moderation, while those of the asset price cycles were becoming more volatile. The need to consider amplitude-free measures of dependence has also been stressed by financial analysts – see, for example, *Sector Performance Across Business Cycles* (2009). Our study also inscribes into the wider discussion concerning the dependence between financial cycles and cycles in the real economy, which recently experiences a revival of popularity (see, for example, Backus et al., 2010; Claessens et al., 2011; Borio, 2014; Égert and Sutherland, 2014, and references therein).