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Robust asset pricing with stochastic hyperbolic discounting

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A B S T R A C T

This paper examines how the interactions of stochastic hyperbolic discounting and ambiguity affect asset pricing. It is found that stochastic hyperbolic discounting has no effects on the equity premium and can raise or lower the risk-free rate, while ambiguity raises the equity premium and always lowers the risk-free rate. Empirical analysis shows that the equity premium puzzle and the risk-free rate puzzle can be resolved by stochastic hyperbolic discounting and ambiguity, while exponential discounting and ambiguity cannot interpret the risk-free rate puzzle.

1. Introduction

The last decade has witnessed extensive research on the effects of dynamically inconsistent preferences on asset pricing. Gong et al. (2007) explore the implications of deterministic hyperbolic discounting for asset prices and return rates. They find that hyperbolic discounting causes the risk-free rate higher than with exponential discounting, while hyperbolic discounting has no effect on the equity premium. Their conclusions show that hyperbolic discounting cannot interpret the equity premium puzzle and deleterious to interpret the risk-free rate puzzle. Palacios-Huerta and Pérez-Kakabadse (2011) preliminarily discuss the implications of stochastic hyperbolic discounting for asset prices but haven't studied the effects of stochastic hyperbolic discounting on the equity premium puzzle and the risk-free rate puzzle.

However, Gong et al. (2007) and Palacios-Huerta and Pérez-Kakabadse (2011) assume that investors have complete confidence in the asset pricing model and do not worry about model uncertainty. Recently, a growing literature begins to concern about the implications of ambiguity (model uncertainty) for asset pricing. Maenhout (2004) employs the robust control approach of Anderson et al. (2003) to analyze asset pricing under ambiguity. He shows that ambiguity increases the equilibrium equity premium and lowers the risk-free rate, and the equity premium puzzle and the risk-free rate puzzle can be resolved by stochastic differential utility and ambiguity. But for the popular CRRA utility, ambiguity can only help interpret the equity premium puzzle and cannot interpret the risk-free rate puzzle. Liu et al. (2005) investigate the relationship between asset pricing and imprecise knowledge about rare events, and show that the option smirk can be explained by ambiguity aversion against rare events. Anderson et al. (2009) show that ambiguity can be a more important determinant

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of equity returns than is risk, which is regarded as volatility in the standard asset pricing. Ju and Miao (2012) obtain various asset pricing implications by developing a generalized recursive smooth ambiguity model. Some other related works include Jang and Park (2016); Jang et al. (2016) and Luo (2016).

In this paper, we extend Palacios-Huerta and Pérez-Kakabadse (2011) to the case of ambiguity along with Maenhout (2004) and explore the joint effects of stochastic hyperbolic discounting and ambiguity on asset pricing. Similar to the case of deterministic hyperbolic discounting in Gong et al. (2007), we show that stochastic hyperbolic discounting has no effects on the equity premium. Unlike the case of deterministic hyperbolic discounting in Gong et al. (2007), we find that the effects of stochastic hyperbolic discounting on the risk-free rate are uncertain. Stochastic hyperbolic discounting can lower the risk-free rate and may help interpret the risk-free rate puzzle. Similar to Maenhout (2004), ambiguity raises the equity premium and always lowers the risk-free rate. Using the estimated parameters in Maenhout (2004); Laibson (1997) and Harris and Laibson (2013), we find that the equity premium puzzle and the risk-free rate puzzle can be resolved by stochastic hyperbolic discounting and ambiguity, while exponential discounting and ambiguity cannot interpret the risk-free rate puzzle.

The rest of the paper is organized as follows. Section 2 presents a robust asset pricing model with stochastic hyperbolic discounting. Section 3 derives the equity premium and the risk-free rate and discusses the effects of stochastic hyperbolic discounting and ambiguity on them. Section 4 calibrates the asset pricing model and interprets the equity premium puzzle and the risk-free rate puzzle. Section 5 offers conclusion.

2. The model

In this section, we characterize a robust asset pricing model with stochastic hyperbolic discounting rather than exponential discounting.

2.1. The basic model

We consider a Lucas-type pure exchange economy with a single good. In this economy, there is a risky stock, S_t, whose dividend D_t follows the following stochastic differential equation:

$$dD_t = \mu_D D_t dt + \sigma_D D_t dB_t, \tag{1}$$

where μ_D and σ_D are the mean growth rate and volatility of the dividend, and B_t is a standard Brownian motion defined on the probability space $\{\Omega, \mathcal{F}_t, \mathbb{P}\}$. The stock price represents a claim on the dividend stream. The total return of the stock consists of both the capital gain and the dividend yield, in that

$$\frac{dS_t + D_t dt}{S_t} = \mu_S dt + \sigma_S dB_t, \tag{2}$$

where μ_S and σ_S are to be determined from equilibrium conditions. There is a locally risk-free bond, which offers a stochastic interest rate r_t that is also to be determined endogenously in equilibrium. Let W_t denote the representative investor's real wealth and ω_t denote the fraction of wealth invested in the risky asset at time t, then the representative investor's wealth constraint is given by

$$dW_t = \{[r + \omega_t(\mu_S - r)]W_t - C_t\}dt + \omega_t \sigma_S W_t dB_t, \tag{3}$$

where C_t is the investor's consumption flow.

The representative investor has time-inconsistent preference. Following Harris and Laibson (2013) and Palacios-Huerta and Pérez-Kakabadse (2011), we assume that the representative investor's discount function is given by

$$h(s) = \begin{cases} e^{-\rho s}, & t \le s < t + \tau_t, \\ \delta e^{-\rho s}, & t + \tau_t \le s < \infty, \end{cases} \tag{4}$$

where $\rho > 0$ and $0 < \delta \le 1$. The discount function decays exponentially at the constant rate ρ up to time $t + \tau_t$, jumps at time $t + \tau_t$ to a multiple δ of its level just prior to $t + \tau_t$, and then decays exponentially at the rate ρ thereafter. We can view $[t, t + \tau_t)$ as the *present* and $[t + \tau_t, \infty)$ as the *future*. When $0 < \delta < 1$, the investor is short-run impatient and he regards the present is more important than the future. When $\delta = 1$, there is no distinction between the present and the future, then we recover Merton's (1969, 1971) exponential discounting. The arrival of the future is stochastic and τ_t is distributed exponentially with parameter $\lambda \ge 0$. Merton's (1969, 1971) exponential discounting model corresponds to the special case $\lambda = 0$ where the future never arrives, while the *instantaneous gratification* (IG) model in Harris and Laibson (2013) corresponds to the limit case $\lambda \to \infty$.

The investor is sophisticated and has time-separable utility with constant relative risk aversion (CRRA). The instantaneous utility function defined on consumption C_t is characterized as

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{5}$$

where $\gamma > 1$ is the relative risk aversion coefficient.