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Dynamic robust portfolio selection with copulas

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ABSTRACT

This paper considers two dynamic robust portfolio optimization models based on the framework of Kakouris and Rustem(2014). We use copula-GARCH and DCC copulas approaches to capture the dynamics of the distribution of the returns. We compare our proposed methods with the static robust and nonrobust portfolio optimization models based on the CSI300 data. The experimental study shows that the dynamic WCVaR models perform better in out-of-sample tests when considering the uncertainty in the estimated model. The static nonrobust method produces higher returns in the in-sample tests, since there is no room to capture model uncertainty.

1. Introduction

The mean-variance (MV) model proposed by Markowitz (1952), lies the foundation of modern portfolio theory, where the portfolio selection problem is formulated as a tradeoff between the return (the portfolio mean) and the risk (the variance). One drawback of the MV model is that it treats both upside and downside deviations from the mean as risk symmetrically, thus it is generally insensitive to extreme events. Value-at-Risk (VaR) is defined as a percentile of a loss distribution, and has been considered as the standard downside risk measure since J.P. Morgan launched RiskMetrics in 1994. VaR can describe more about extreme events. However, VaR is not subadditive, therefore, it is not a coherent risk measure in the sense of Artzner et al. (1999). Another shortcoming of VaR is that it is just a percentile of a loss distribution, so it provides no information of the extreme losses beyond VaR. For this purpose, Conditional Value-at-Risk (CVaR), defined as the mean of the tail distribution exceeding VaR, has become popular in financial risk management. Rockafellar and Uryasev (2000), 2002) propose a linear formulation of CVaR, which can be solved efficiently. Acerbi and Tasche (2002) prove that CVaR is a coherent risk measure.

Black and Litterman (1992) point that the portfolio policy is more sensitive to the mean than to the covariance matrix when implementing the mean-variance strategy. Moreover, a small uncertainty in the mean vector can make the usual optimal solution of the problem completely meaningless from a practical viewpoint. Robust optimization is one of the effective methods to deal with data uncertainty. It assumes that the information on the underlying probability distribution is only partial known. Ghaoui et al. (2003) study the worst-case VaR (WVaR) where only bounds on the mean and covariance matrix are available. They show that these problems can be formulated as semidefinite programs. Zhu and Fukushima (2009) consider the worst-case CVaR (WCVaR) under mixture distribution uncertainty, box uncertainty, and ellipsoidal uncertainty, the corresponding problems are cast as linear programs and second-order cone programs. Li et al. (2016) construct an asymmetry robust mean absolute deviation (ARMAD) model that takes the asymmetry distribution of returns into consideration.

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Computational experiments show that the ARMAD method can distinguish the high return stocks. Hu (2002, 2006) construct a mixture copula model so that it can capture various patterns of dependence structures. In a mixed copula, the degree of dependence is carried via the association parameters, and the shape of the dependence is summarized by the weight on each individual copula function. Inspired by the work of Hu (2002, 2006) and Zhu and Fukushima (2009), Kakouris and Rustem (2014) demonstrate one way of using copulas in a portfolio optimization framework and derive the copula formulation of CVaR and WCVaR. They study the WCVaR under mixture copula distribution uncertainty, the corresponding problem is formulated as a linear programming problem. They use Gaussian copula and three Archimedian copulas in the mixture structure, which can cover a large spectrum of possible dependencies.

Most of the related literature is based on static copulas, among which static Gaussian copulas are especially common (e.g., Li, 2000). However, many empirical studies show that the correlation between equity returns is asymmetric, nonlinear, and time-varying (Erb et al., 1994; Longin and Solnik, 1995; Tse, 2000; Longin and Solnik, 2001; Ang and Chen, 2002). Engle (2002); Tse and Tsui (2002) introduce multivariate GARCH models with time-varying correlations simultaneously. Since correlations have been found to be time-varying, it is unreasonable to treat copula dependence parameters as constant. To our knowledge, Patton (2006) is the first to propose time-varying copula, they consider the change of copula parameters by assuming that the dependence measure is a function of the conditional volatilities of the underlying financial variables. The author used this time-varying copula method to model the dependence structure between two exchange rates and showed that the time-varying copula model behaves better than the static one. Heinen and Valdesogo (2008) extend the DCC–GARCH approach of Engle (2002) to estimate the dynamics of non-elliptical copulas parameters. For a recent survey on time varying copulas see Manner and Reznikova (2012). Manner and Reznikova (2012) show that the DCC copula model of Heinen and Valdesogo (2008) performs quite well and is easy to implement, therefore, it can be used in many situations. Weiß (2013) compare the accuracy of the copula-GARCH and Dynamic Conditional Correlation (DCC) models for forecasting the Value at Risk (VaR) and Conditional Value at Risk (CVaR) of bivariate portfolios. Berger (2013) forecast the VaR and combine elliptical copulas with time varying DCC matrices and Extreme Value Theory (EVT) based models for the marginal return distributions. Li and Li (2015) use a type of dynamic copula method to capture the dependence structure between financial assets and price basket default swaps.

In this paper, we extend the static copula formulation of WCVaR of Kakouris and Rustem (2014) to dynamic robust portfolio optimization models. We use the DCC copulas model of Heinen and Valdesogo (2008) and copula-GARCH model to forecast the WCVaR of bivariate portfolios.

Our contributions can be summarized as follows: First, we construct two dynamic robust portfolio optimization models. Moreover, the dynamic robust models can be formulated as linear programming problems, which can be solved efficiently. Second, we study the in-sample and out-of-sample analysis of the dynamic robust, static robust and static nonrobust strategies. The empirical results show that the dynamic robust approaches are able to distinguish the assets with low volatility and perform the best for the out-of-sample tests. While the static nonrobust method produces higher returns in the in-sample tests, since evaluating performance on the same data used to estimate a model leaves no room to capture model uncertainty. The robust rules lead to less aggressive trading and can provide investors with good advice.

The remainder is organized as follows: In Section 2, we briefly present the DCC copulas model considered in this article. Section 3 contains an introduction to the static copula formulation of WCVaR of Kakouris and Rustem (2014). We introduce the procedure to forecast portfolio WCVaR in Section 4. Section 5 focuses on the empirical analysis of the dynamic robust CVaR models using Chinese real market data. Section 6 gives the conclusion.

Notations. $\boldsymbol{z}^{\boldsymbol{T}}$ and A^T denote vector transpose and matrix transpose, respectively. We use boldface letters to denote vectors and capital letters to denote matrices.

2. Robust CVar model of Kakouris and Rustem (2014)

In this section, we review the copula formulation for WCVaR of Kakouris and Rustem (2014).

Let $f(\mathbf{w}, \mathbf{x})$ denote the loss associated with the decision vector $\mathbf{w} \in \mathbb{R}^n$ and the random return vector $\mathbf{x} \in \mathbb{R}^n$. The vector $\mathbf{w} = (w_1, \dots, w_n)$ denotes the weight of investment in each asset. We assume that $\mathbf{x}$ follows a continuous distribution with density function p(.).

Given a decision vector $\mathbf{w} \in \mathbb{W}$, the probability of $f(\mathbf{w}, \mathbf{x})$ not exceeding a threshold α is represented as

$$\Psi(\mathbf{w}, \alpha) \triangleq \int_{f(\mathbf{w},\mathbf{x}) \le \alpha} p(x)dx.$$

The VaR and CVaR for the loss associated with a fixed $\mathbf{w} \in \mathbb{W}$ and a confidence level β are denoted by $VaR_\beta(\mathbf{w})$ and $CVaR_\beta(\mathbf{w})$. They are defined as

$$VaR_\beta(\mathbf{w}) \triangleq \min\{\alpha \in \mathbb{R} : \Psi(\mathbf{w}, \alpha) \ge \beta\}.$$

$$CVaR_\beta(\mathbf{w}) \triangleq \frac{1}{1-\beta} \int_{f(\mathbf{w},\mathbf{x}) \ge VaR_\beta(\mathbf{w})} f(\mathbf{w}, \mathbf{x}) p(x) dx.$$