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Multi-period portfolio optimization under probabilistic risk measure

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ABSTRACT

This paper develops a minimax model for a multi-period portfolio selection problem. An analytical solution is obtained and numerical simulations demonstrate the superiority of the multi-period model over its corresponding single period one, as well as over the market index.

1. Introduction

The measure of risk is of great importance in portfolio management, especially when the distribution of the portfolio returns are nonsymmetric (Leland, 1999; Pedersen, 2001) and investors are averse to downside loss (Ang et al., 2006; Bali et al., 2009; Kahneman and Tversky, 1979). Researchers proposed some quantile-based risk measures in the past decades. Among these measures, Value-at-Risk (VaR) and Conditional VaR (CVaR) attract much attention in both academy and practice (Duffie and Pan, 1997; Jorion, 2007; Linsmeier and Pearson, 2000; Rockafellar and Uryasev, 2000; 2002). More recently, Sun et al. introduced a probabilistic risk measure, with allowance to cater for investors with different degree of risk aversion (Sun et al., 2015).

It is at least equally important to embed these risk measures into portfolio management techniques although it is not an easy task to do so numerically. Basak and Shapiro theoretically compared the portfolio optimization strategies of the LEL (Limited-Expected-Losses) and VaR risk managers in a general equilibrium framework (Basak and Shapiro, 2001). Alexander and Baptista compared the VaR and CVaR constraints on portfolio selection (Alexander and Baptista, 2004). Brandtner modeled the mean-spectral risk preferences in a form of spectral utility function (Brandtner, 2013). Some other researchers proposed some novel methods to search for optimal portfolios with risk measures for downside loss-averse preferences (Cui et al., 2013; Jarrow and Zhao, 2006; Roman et al., 2007; Sengupta and Sahoo, 2013; Yao et al., 2013). However, most of

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these numerical studies implemented Monte Carlo simulation to search for optimal portfolios, and could not find an analytical solution for the optimization problem. A computationally simple analytical solution is highly useful, especially for unsophisticated investors. Sun et al. stood out in the literature by providing a model with an analytical solution (Sun et al., 2015).

This paper extends the work of Sun et al. (2015) to the multi-period setting. In their paper, the authors construct a minimax portfolio selection model by introducing a probabilistic risk measure, and a analytical solution is computationally available. Their paper aimed to maximize the expected portfolio return and minimize the maximum individual risk of the assets in the portfolio for a single period. However, portfolios are dynamically managed over multiple periods in practice. Some other papers also argued for the importance of the dynamic relationship between risk and return in a long horizon (Bali et al., 2009; Bickel, 1969; Harrison and Zhang, 1999; Merton, 1973).

Our paper contributes to the literature in the following way. Firstly, we provide a computationally simple analytical solution to the complex portfolio selection problem in a multi-period setting. Secondly, our model is superior to the Sun et al. model which is superior to others in the mean-variance space (Sun et al., 2015). Thirdly, out model inherits some good features of the single-period model. For example, we are still able to derive an analytical solution without computation of the covariances. In addition, the investors do not have to purchase a huge number of stocks to form an optimal portfolio.

The remainder of this paper is organized as follows. Section 2 describes the problem and formulates it as a bi-criteria optimization problem. Section 3 develops the analytical solution to the problem. In Section 4, ASX100 data is applied to our model and to the single period model, and the results compared. Section 5 concludes the paper.

2. Problem formulation

We consider a multi-period portfolio optimization problem, where an investor is going to invest in N possible risky assets $S_j,\ j=1,\dots,N$ with a positive initial wealth of M_0. The investment will be made at the beginning of the first period of a T-period portfolio planning horizon. Then, the wealth will be reallocated to these N risky assets at the beginning of the following $T-1$ consecutive time periods. The investor will claim the final wealth at the end of the Tth period.

Let x_{tj} be the percentage of wealth at the end of period $t-1$ invested in asset S_j at the beginning of period t. Denote $\boldsymbol{x}_t=[x_{t1},\dots,x_{tN}]^\top$. Here we assume that the whole investment process is a self-financing process. Thus, the investor will not increase the investment nor put aside fund in any period in the portfolio planning horizon. In other words, the total fund in the portfolio at the end of period $t-1$ will be allocated to those risky assets at the beginning of period t. Thus,

$$\sum_{j=1}^{N} x_{tj}=1,\quad t=1,\dots,T. \tag{2.1}$$

Moreover, it is assumed that short selling of the risky assets is not allowed at any time. Hence, we have

$$x_{tj}\geq 0,\quad t=1,\dots,T,\ j=1,\dots,N. \tag{2.2}$$

Let R_{tj} denote the rate of return of asset S_j for period t. Define $\boldsymbol{R}_t=[R_{t1},\dots,R_{tN}]^\top$. Here, R_{tj} is assumed to follow normal distribution with mean r_{tj} and standard deviation σ_{tj}. We further assume that vectors $\boldsymbol{R}_t$, $t=1,\dots,T$, are statistically independent, and the mean $E(\boldsymbol{R}_t)=\boldsymbol{r}_t=[r_{t1},\dots,r_{tN}]^\top$ is calculated by averaging the returns over a fixed window of time τ.

Let

$$r_{tj}=\frac{1}{\tau}\sum_{i=t-\tau}^{t-1} R_{ji},\quad t=1,\dots,T,\ j=1,\dots,N. \tag{2.3}$$

We assume that in any time period, there are no two distinct assets in the portfolio that have the same level of expected return as well as standard deviation, i.e., for any $1\leq t\leq T$, there exist no i and j such that $i\neq j$, but $r_{ti}=r_{tj}$, and $\sigma_{ti}=\sigma_{tj}$.

Let V_t denote the total wealth of the investor at the end of period t. Clearly, we have

$$V_t=V_{t-1}(1+\boldsymbol{R}_t^\top\boldsymbol{x}_t),\quad t=1,\dots,T, \tag{2.4}$$

with $V_0=M_0$.

First recall the definition of probabilistic risk measure, which was introduced in Sun et al. (2015) for the single period probabilistic risk measure.

$$w_p(\boldsymbol{x})=\min_{1\leq j\leq N}\Pr\{\,|R_jx_j-r_jx_j|\leq\theta\varepsilon\,\}, \tag{2.5}$$

where θ is a constant to adjust the risk level, and ε denotes the average risk of the entire portfolio, which is calibrated by the function below.

$$\varepsilon=\frac{1}{N}\sum_{j=1}^{N}\sigma_j. \tag{2.6}$$

The whole idea of this risk measure in (2.5) is to locate the single asset with greatest deviation in the portfolio. With this 'biggest risk' mitigated, the risk of the whole portfolio can be substantially reduced as well. For multi-period portfolio