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Ambiguity and optimal portfolio choice with Value-at-Risk constraint

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ABSTRACT

Integrating a Value-at-Risk constraint on a fund manager's wealth and ambiguity, we present a model of optimal portfolio choice for a fund manager who allocates her wealth between risky and riskless assets. When a fund manager controls asset composition, her reactions differ with respect to an increase in only risk aversion and only ambiguity aversion. When the sum of coefficients of risk aversion and ambiguity aversion is fixed, the effect of risk aversion on risky investment dominates the effect of ambiguity aversion in that stock holdings are dramatically smaller in the absence of ambiguity aversion than in its presence.

1. Introduction

Models of dynamic portfolio choice between risky and riskless assets have helped us to understand the testable empirical predictions between asset pricing and consequent portfolio demand, and have been building blocks for testable general equilibrium models. Since the seminal paper of Merton (1969, 1971), a plethora of studies have investigated the optimal asset composition in a dynamic environment, however, their results have failed to make their close relatives attractive prospects for use in practice.[2]

Along with this line, we present a model of optimal portfolio choice that integrates two main motives that have been identified as quantitatively so important in asset pricing, portfolio choice, and recently corporate finance. First, a portfolio

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[2] For example, not withstanding a very important empirical observation of moderate equity holdings for investors, the counterfactual results that produced a significant amount of portfolio share have been obtained from the existing literature. Further, in the population as a whole, stock market participation rates have been very low, nevertheless it was optimal for an investor to invest in the stock market (Merton, 1969, 1971). According to Gomes and Michaelides (2005), portfolio share was significantly lower than 100% and the stock market participation rate in the U.S. population was about 50%.

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constraint is driven by the presence of a Value-at-Risk (VaR) constraint.[3] Second, ambiguity (or model uncertainty) about equity returns is taken into account when studying optimal portfolio choice. The combination of portfolio constraints and ambiguity can be incrementally attributed exclusively to the financial literature on dynamic portfolio decisions. We hope that some results of this paper will lend themselves to the study of implications for risk management and portfolio choice in practice.

The bulk of literature on dynamic portfolio choice incorporates some of these motives. Models of portfolio choice have been extended to include portfolio constraints that preclude wealth from falling below some prespecified level (Grossman and Vila, 1989; Basak, 1995; Grossman and Zhou, 1996).[4] In addition, Basak and Shapiro (2001) yield testable predictions about optimal behavior consistent with Value-at-Risk-based risk management. More recently, different from existing literature, Dai et al. (2011) explore the joint impact of position limits of investors and transaction costs incurred when buying and selling stocks.

Understanding the implications of investors' concern about model uncertainty has been so important in finance and hence the effects of *uncertainty* have also been widely studied through a concept of *ambiguity aversion*.[5] Since the seminal paper of Hansen and Sargent (1995, 2001) and Hansen et al. (2006) extend the model of Gilboa and Schmeidler (1989), who axiomatize max-min utility, to a continuous-time robust consumption and portfolio choice model. Epstein and Scheneider (2003) derive a recursive model for utility building on an intertemporal model of multiple-priors utility. Garlappi et al. (2007) formulate an ambiguity-averse fund manager's portfolio selection problem within the standard mean-variance portfolio optimization. Anderson et al. (2009) measure the uncertainty as the degree of disagreement of professional forecasters concerning the returns. They derive an uncertainty-return trade-off and show that uncertainty can be a more important determinant of equity returns than is risk, which is regarded as volatility in the standard asset pricing.

The study of robustness in optimal portfolio choice is economically significant, but the robustness itself contributes to resolution of various financial puzzles. Uppal and Wang (2003) show that ambiguity leads to a portfolio that substantially underdiversified relative to the standard mean-variance portfolio. Maenhout (2004) provides an alternative explanation for the equity premium puzzle by incorporating homothetic robustness. Liu et al. (2005) investigate the relationship between asset pricing and imprecise knowledge about rare events, and assert that the option smirk can be explained by ambiguity aversion against rare events. Recently, Ju and Miao (2012) obtain various asset pricing implications by developing a generalized recursive smooth ambiguity model: the procyclical pattern of price-dividend ratios, the counter-cyclical pattern of equity premia and equity volatility, the leverage effect, and the mean reversion of excess returns.

Integrating a VaR constraint on a fund manager's terminal wealth and ambiguity (or model uncertainty), we present a model of optimal portfolio choice for a fund manager who has to allocate her wealth between risky and riskless assets. To examine the effects of varying coefficients of risk and ambiguity aversions in the simplest possible environment, we consider a financial market with a constant investment opportunity set in a partial equilibrium. We try to devote our attention on investigating how optimal asset composition of a fund manager is affected by the effects of risk and ambiguity aversions, under a VaR constraint on her final wealth. We believe our paper is the first to integrate ambiguity into the portfolio choice while taking into account a realistic portfolio constraint such as the VaR constraint which is a standard choice by industry regulations.

Our end state is to complement two important articles by solving a fund manager's problem explicitly and by obtaining results numerically for the more general case of optimal portfolio choice. Basak and Shapiro (2001) study an optimal portfolio choice with a VaR constraint and conclude that the VaR constraint often leads to fund managers' aggressive stock investment. Maenhout (2004) takes ambiguity about equity returns into account when studying optimal portfolio decisions and asserts that the effects of ambiguity significantly reduce the demand for equities. We show that the aggressive investment due to the consideration of the VaR constraint is true in the absence of the effects of ambiguity. However, in the presence of ambiguity, the effect of the VaR constraint is mitigated.

One can argue that as in Maenhout (2004), ambiguity aversion raises the level of risk aversion by the amount of ambiguity aversion and hence that the effect of ambiguity aversion on a fund manager's asset composition is trivial, even under a VaR constraint on her terminal wealth. However, when a fund manager controls asset composition, her reactions differ with

[3] VaR is the lowest tail percentile for losses from the distribution of profit and loss. VaR-based risk management has been a popular standard choice by industry regulations (Jorion, 2006). Fund managers and financial institutions use VaR as the relevant risk measure (Hull, 2005). Whether VaR is an appropriate risk measure is still debatable (Basak and Shapiro, 2001; Berkowitz and O'brien, 2002).

[4] A number of researchers have showed that portfolio adjustments are significantly affected by various portfolio constraints. For the details, see Cvitanić and Karatzas (1999), Cuoco (1997), and Cuoco and Liu (2000).

[5] Importantly, ambiguity and risk must be distinguished from the point of view of probabilized and non-probabilized events, respectively. Ambiguity refers to an attitude of preference for known risks over unknown risks in decision theory. If a decision maker knows an appropriate probability distribution to guide choice, then ambiguity is a known risk. However, when a decision maker is concerned about an unknown probability distribution, we classify ambiguity as an unknown risk. This type of ambiguity has been widely explored by economists Ellsberg (1961); Kahneman and Tversky (1979); Tversky and kahneman (1992). On the other hand, Knight (1921) and Keynes (1936) show that decision makers are ambiguity averse, i.e., they have Knightian uncertainty. Note that model uncertainty is equivalent to Knightian uncertainty when decision makers do not know an exact probability distribution. This is why we sometimes consider the interchangeable use of ambiguity and model uncertainty. However, model uncertainty is to some extent different to Knightian uncertainty and this difference should be taken into account when studying ambiguity (or model uncertainty) (Guidolin and Rinaldi, 2013). In this paper, ambiguity (or model uncertainty) is considered as a particular type of model misspecification following Maenhout (2004), i.e., uncertainty about return processes.