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Model misspecification and pricing of illiquid claims☆

Alexey Rubtsov

Department of Mathematics, Ryerson University, Victoria Building, 350 Victoria Street, Toronto, Ontario, Canada

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ABSTRACT

This paper analyses the impact of model misspecification on pricing and hedging of illiquid claims. We consider the case when an ambiguous investor hedges his position in an illiquid claim, written on a nontraded asset, by investing in a tradable asset. The optimal trading strategy and utility indifference price of the claim are derived. It is shown that when the model for the underlying asset is misspecified, the utility indifference price is not necessarily increasing or decreasing in the correlation between traded and nontraded assets. An explanation for the puzzle of why small retail investors buy structured bonds is given.

1. Introduction

Classical option pricing theory assumes that markets are complete and that the underlying assets are liquid securities with known probability distributions. However, as pointed out in Davis (2006); Jensen and Jørgensen (2012) and other numerous studies, the liquidity and completeness assumptions are frequently violated. Furthermore, the experimental studies of Ellsberg (1961) and Bossaerts et al. (2010) show that individuals are not only averse to risk (known probability distribution), but also averse to ambiguity (unknown probability distribution). In this paper we solve the problem of an illiquid claim valuation for a risk- and ambiguity-averse investor.

We consider an investor who has a reference model for prices of traded and nontraded assets, but realizes that other models might represent reality better. Hence, he prefers investment rules that are robust to model misspecification by performing reasonably well across a set of plausible models. The discrepancy between the reference model and alternative models is defined in terms of relative entropy which serves as a penalty in the optimization procedure. This penalty quantifies the investor's uncertainty aversion about the reference model. The utility indifference price of an illiquid claim is derived in closed form after solving the relevant robust Hamilton–Jacobi–Bellman equation, cf. (Anderson et al., 2003). We demonstrate that adding ambiguity can have a significant effect on investors' decision-making.

This paper makes the following four contributions. First, we show that the utility indifference price of an illiquid claim can be decreasing in correlation between traded and nontraded assets which is not true in absence of ambiguity (see Henderson, 2002) because usually the higher the correlation the better hedge can be achieved. Second, assuming that the trader is ambiguous about the nontraded asset only, we demonstrate that the speculative component of the investor's optimal portfolio is not influenced by ambiguity. Third, in a numerical example we show that the correlation between traded

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E-mail address: arubtsov@ryerson.ca, alexeyvrubtsov@gmail.com

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and nontraded assets is more important for a less ambiguous investor. Intuitively, the correlation becomes less informative for hedging when a trader has large uncertainty about the model for the underlying.

The fourth contribution of our paper is the application of utility indifference pricing under ambiguity to structured bonds pricing. It is argued in Bernard et al. (2009); Henderson and Pearson (2011) and Célérier and Valée (2013) that small investors pay more for structured products than the actual product value. Jensen and Jørgensen (2012) try to resolve this puzzle. As a structured bond they consider a combination of a zero-coupon bond and an option. In particular, they show that investors should include structured bonds in their optimal portfolio if they cannot access the option underlying directly and if the products provide sufficient diversification to compensate for their costs. Following the setup of Jensen and Jørgensen (2012) we provide another explanation for the puzzle of why small retail investors buy structured bonds. We find that the more the investor is ambiguous about the option underlying, the smaller price he is willing to pay for the structured bond. Therefore, high price that is actually paid for the bonds can be due to the fact that retail investors underestimate their actual uncertainty about the products.

The ability of financial institutions to induce mistakes of judgement and/or obscure the properties of the products has been studied in the literature. Piccione and Spiegler (2012) develop the model where firms can influence consumer' ability to compare market alternatives. Sato (2014) reports that opacity price premium (paying more than the fair value) incentivizes financial engineers to render transparent assets opaque deliberately. Henderson and Pearson (2011) claim that financial institutions can take advantage of investors' mistakes about assigning probabilities to certain events in the market.[1] Thus, retail investors can underestimate their uncertainty about structured bonds due to (possibly deliberate) complexity of the products.

This paper is related to the literature on model uncertainty, ambiguity aversion, and illiquid claim valuation. We apply the robust control theory developed in Anderson et al. (2003). Maenhout (2004); 2006) obtains portfolio rules for an ambiguous investor with power utility function in a setting with constant investment opportunities and in a setting with a mean-reverting risk premium, respectively. Uppal and Wang (2003) study intertemporal portfolio choice when the investor allows for ambiguity about joint distribution of stock returns and for different levels of ambiguity for the marginal distribution of returns. Liu (2010) extends Maenhout's work further to Epstein–Zin preferences and Liu (2011) assumes a regime-switching expected stock return. Flor and Larsen (2014) study the effect of ambiguity aversion on the optimal bond-stock-cash allocation when the interest rates are stochastic. Munk and Rubtsov (2014) solve the portfolio choice problem for a risk- and ambiguity-averse investor when interest rates and the inflation rate are stochastic.

The problem of pricing claims on nontraded assets in discrete time is tackled in Smith and Nau (1995); Detemple and Sundaresan (1999); Musiela and Zariphopoulou (2004) where a second, correlated asset is available for trading. (Duffie and Richardson, 1991) solve a continuous-time problem of determining the optimal futures hedging policy for a commitment in nontraded asset under the assumption of quadratic utility. A utility indifference price under exponential utility was studied by Tepla (2000) where the illiquid asset payoff is normally distributed. Henderson and Hobson (2002) and Henderson (2002) obtained the utility indifference price of a claim which is a function of the nontraded asset. Musiela and Zariphopoulou (2004) considered the case of an arbitrary diffusion process for the nontraded asset.

The paper is organized as follows. In Section 2 the problem of utility indifference pricing under model misspecification is formulated. The price of an illiquid claim for an ambiguous investor is derived in Section 3. The application of utility indifference pricing to structured bonds pricing is done in Section 4. Section 5 summarizes the results. The proofs of some results are given in the appendix.

2. Problem formulation

In this section we formulate the problem of an illiquid claim valuation for an investor who cannot trade in the underlying asset. To hedge his position in the claim, the investor uses a traded asset that has a high correlation with the nontraded underlying asset.

The prices of the nontraded (S_t) and the traded (P_t) assets follow the dynamics

$$dS_t = S_t(\mu_S dt + \sigma_S dB_{St}), \tag{2.1}$$

$$dP_t = P_t\left(\mu_P dt + \sigma_P(\rho dB_{St} + \sqrt{1-\rho^2}dB_{Pt})\right), \tag{2.2}$$

respectively, where the constants μ_S, μ_P are expected returns, $\sigma_S, \sigma_P > 0$ are returns volatilities, and B_{St}, B_{Pt} are independent standard Brownian motions.

It is also assumed that the investor holds λ shares of an illiquid claim with the payoff $h(S_T)$ which he is not able to sell until time T. Therefore, the agent's wealth at time T is determined by his investments in the traded asset and by the claim's payoff. If $|\rho| < 1$, then the market is incomplete because there is a nontraded asset and therefore, we cannot perfectly hedge and uniquely price the claims written on the asset. In this respect, the hedging strategy and the price of the claim will depend on the agent's utility function.

[1] In this respect it should be noticed that uncertainty about probability distributions is explicitly incorporated in our model.