Computers & Geosciences 74 (2015) 87–96

Contents lists available at ScienceDirect

Computers & Geosciences

journal homepage: www.elsevier.com/locate/cageo

Inflow–outflow boundary conditions along arbitrary directions in Cartesian lake models

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ARTICLE INFO

Article history:
Received 7 March 2014
Received in revised form
6 October 2014
Accepted 7 October 2014
Available online 30 October 2014

Keywords:
Three dimensional (3D) simulation
Cartesian grid
Inflow angle
Source–sink cell
Mixing
Lake

ABSTRACT

Specifying point sources and sinks of water near boundaries is presented as a flexible approach to prescribe inflows and outflows along arbitrary directions in Cartesian grid lake models. Implementing the approach involves a straightforward modification of the governing equations, to include a first order source term in the continuity and momentum equations. The approach is implemented in a Cartesian grid model and applied to several test cases. First, the flow along a straight flat bottom channel with its axis forming different angles with the grid directions is simulated and the results are compared against well-known analytical solutions. Point-sources are then used to simulate unconfined inflows into a reservoir (a small river entering a reservoir in a jet-like manner), which occur at an angle with the grid directions. The model results are assessed in terms of a mixing ratio between lake and river water, evaluated at a cross section downstream of the inflow boundary. Those results are particularly sensitive to changes in the inflow angle. It is argued that differences in mixing rates near the inflow sections could affect the fate of river-borne substances in model simulations.

1. Introduction

The space–time distribution of particulate and dissolved substances in lakes and reservoirs, the light and nutrient availability for algal growth and, in general, the environment in which biogeochemical reactions occur are largely controlled by transport and mixing processes in the water column. Describing and understanding the physical processes leading to mixing and transport in the water column, hence, is the first step that needs to be taken to understand the chemical and biological properties of aquatic ecosystems, and its spatial and temporal variability. To this end, considerable efforts have been devoted during the last few years to develop and apply numerical models, capable of solving the governing equations of fluid motion and, hence, describing the flow environment in three-dimensions with a high temporal and spatial resolution and low computational cost. Most of these large-scale flow models are based on the solution of the three-dimensional form of the shallow-water equations 3D-SWE, subject to the appropriate boundary conditions. The correct representation of the specific flow patterns that develop in any given water body depends mainly on the ability of the model to represent accurately the mass and energy fluxes (their frequency, intensity, duration and timing) that occur through the free surface – and which are the drivers of motion in the water column – and the morphometry of the system (Imboden and Wüest, 1995). This, in turn, largely depends on how the physical space is discretized on the model grid (grid system). The most widely used grid system in 3D lake modeling is the Cartesian-grid (e.g. Hodges et al., 2000; Rueda et al., 2003; Appt et al., 2004; Laval et al., 2005; Okely and Imberger, 2007; Hoyer et al., 2014). Model coding and grid definition in this grid-system is much simpler than in others. Grid generation, for example, in unstructured-grid models is not a completely automatic process, requiring separate grid creation software, and user intervention is often need to produce a grid of satisfactory quality (Liang et al., 2007), especially if complex topographic features are present. It is also computationally expensive.

In spite of their simplicity, Cartesian grid lake models tend to produce locally inaccurate solutions where the shoreline is not aligned with the Cartesian grid directions and is represented as a staircase. A variety of approaches have been proposed to resolve correctly the near shore circulation. The grid resolution can be increased near the shoreline, for example, using 'plaid' structured meshes (i.e. non-uniform Cartesian grid spacing), adaptive mesh refinements or nested grids (e.g. Berger and Oliger, 1984; Ham et al., 2002; Gibou et al., 2007; Peng et al., 2010, and references therein). Cut cells can also be used for the solution of the shallow water equations (Causon et al., 2000; Liang et al., 2007), and in this case, boundary contours are cut out of a background Cartesian mesh and cells that are partially or completely cut are singled out for special treatment. Other approaches such as the immerse boundary method of Peskin (1972, 2002), the virtual

Abbreviations: ; 3D-SWE, three-dimensional shallow water equations; SC, sources and sinks; NF, normal velocity component along faces; SML, surface mixed layer

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http://dx.doi.org/10.1016/j.cageo.2014.10.002
0098-3004/

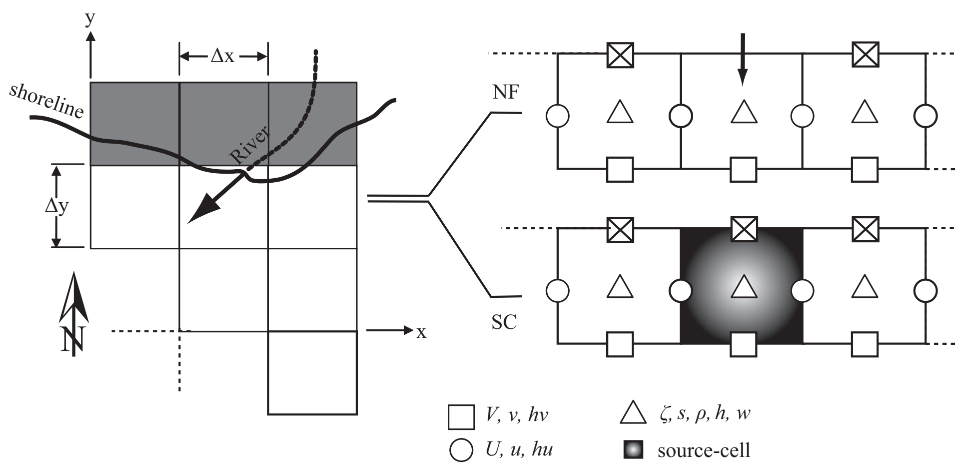

Fig. 1. (Left) Schematic plot illustrating the entrance of river inflows at an angle with the Cartesian grid and (right) how these river inflows would be specified with the (top right) NF-method and the (bottom right) SC-method. Black arrows show the direction of the real inflows (left) and those prescribed with the NF-method (top right). Squares, circles and triangles show where variables are defined within a given cell. Crossed symbols show the defined variable is set to zero.

boundary method (Saiki and Biringen, 1996) or the Brinkman penalization method (e.g. Reckinger et al., 2012) introduce a source (force) term in the momentum equations, to represent the force exerted by solid boundaries on the fluid.

An additional problem arising from the Cartesian representation of lake boundaries is related to the simulation of river inflows and outflows, which may not be aligned with the grid directions (Fig. 1). Flow boundary conditions (clamped boundary conditions) are typically prescribed in lake models (e.g. Smith, 2006; Hodges et al., 2000) by setting the values of the velocity components normal to the grid directions at the faces of the boundary cells (Fig. 1). Flow directionality with this approach, which will be referred to as NF-method (for normal velocity component along faces), could be wrong. The effects of inflows on circulation and mixing – whether these effects are localized (Rueda and Vidal, 2009) or if they impact the basin-scale motions (Hollan, 1998) – or the fate of river-borne substances, may not be correctly simulated with the NF-method. Our goal is to present an alternative approach to specifying inflow and outflow boundary conditions in Cartesian lake models, in which flow direction is independent of grid alignment. It consists of using point sources and sinks of mass and momentum in grid cells which are next to solid boundaries, where water is added or detracted from the computational domain (Fig. 1). This approach, here referred to as SC (for sources and sinks), implies a simple-to-implement modification of the governing equations. The grid, in turn, does not need to be modified. The use of sources and sinks of mass and momentum has been successfully applied in the lake modeling literature (Singleton et al., 2010) to simulate the effect of bubble-plumes on lake circulation, and, hence, on hypolimnetic oxygen and density fields. Here, the method is adapted to represent the effect of localized flows into and out of the domain, with length scales which are well below the grid resolution of the model. It is examined whether ignoring the directionality of inflows may affect or not the results of local and larger basin-scale simulations of mixing and transport in lakes and reservoirs.

2. Methods

2.1. Approach

The SC and the NF approaches to specifying flow boundaries in a 3D-SWE model will be first described. These two approaches were compared in a test case in which the flow boundaries are aligned with the grid directions. The test consists on the simulations of the flow field along a straight rectangular channel with flat bottom laid out along the x-axis. The SC-method will be then applied to the same straight channel, but in this case, the channel will be assumed to form an angle with the Cartesian grid directions. The SC-method will be then applied to simulate environmental flows in a lake in which the use of boundaries not aligned to the Cartesian grids are needed.

2.2. Governing equations with point sources and sinks of fluid

Assuming that (1) variations in density are negligible everywhere except in the buoyancy term (the Boussinesq approximation), (2) the weight of the fluid balances the pressure in the equation for vertical momentum (the hydrostatic approximation), and (3) a diffusion-like term can be used to represent turbulent fluxes of scalars and momentum (the eddy diffusivity concept), the Navier–Stokes equations, incorporating point sources and sinks of fluids, can be written as (adapted from the work of Lynch, 1986):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = \frac{\delta}{\rho_0} \tag{1}$$

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x}\left[\int_{-D}^{\zeta} u\, dz\right] + \frac{\partial}{\partial y}\left[\int_{-D}^{\zeta} v\, dz\right] = \int_{-D}^{\zeta} \frac{\delta}{\rho_0} dz \tag{2}$$

$$\begin{aligned}\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} - fv \\ = -\left(g\frac{\partial \zeta}{\partial x} + g\frac{1}{\rho_0}\int_z^{\zeta} \frac{\partial \rho}{\partial x} dz'\right) + \frac{\partial}{\partial x}\left(A_h\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_h\frac{\partial u}{\partial y}\right) \\ + \frac{\partial}{\partial z}\left(A_v\frac{\partial u}{\partial z}\right) + \frac{\delta}{\rho_0}(u - u_0)\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + fu \\ = -\left(g\frac{\partial \zeta}{\partial y} + g\frac{1}{\rho_0}\int_z^{\zeta} \frac{\partial \rho}{\partial y} dz'\right) + \frac{\partial}{\partial x}\left(A_h\frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(A_h\frac{\partial v}{\partial y}\right) \\ + \frac{\partial}{\partial z}\left(A_v\frac{\partial v}{\partial z}\right) + \frac{\delta}{\rho_0}(v - v_0)\end{aligned} \tag{4}$$