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Portfolio optimization using asymmetry robust mean absolute deviation model

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ABSTRACT

In this paper, we construct an asymmetry robust mean absolute deviation (ARMAD) model that takes the asymmetry distribution of returns into consideration. We test different robust strategies using the historical data of Chinese small cap stocks based on the growing and declining market, respectively. Computational experiments show that the ARMAD method can distinguish the high return stocks. Since there is short-run persistence of relative performance of the stocks, the portfolios constructed by the ARMAD model can provide investors with good guidance in the near future.

1. Introduction

The mean-variance (MV) model proposed by Markowitz (1952), lies the foundation of modern portfolio theory, where the portfolio selection problem is formulated as a tradeoff between the return (the portfolio mean) and the risk (the variance). Since the MV model is a quadratic programming problem and the size of the covariance matrix could be very large and difficult to estimate (Konno and Yamazaki, 1991) propose a linear programming model based on the mean absolute deviation (MAD) risk function to solve large-scale problems on a real time basis. The other drawback is that the MV model treats both upside and downside deviations from the mean as risk symmetrically, which results in unfavourable asset allocations when the returns are asymmetrically distributed.

In the MAD model, the uncertain returns are replaced by the expected values, which are typically estimated from the historical data. As pointed by Black and Litterman (1992), when implementing the mean-variance strategy, the portfolio policy is more sensitive to the mean than to the covariance matrix. Moreover, a small uncertainty in the mean vector can make the usual optimal solution of the problem completely meaningless from a practical viewpoint. Robust optimization is one of the effective methods to deal with data uncertainty. It does not assume that the probability distributions are known. Instead, the uncertain data are just assumed to lie in a so-called uncertainty set. The first robust linear optimization model is due to Soyster (1973), where the uncertainty set includes every possible realization of the uncertain parameters such that the

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probability of violating the constraint is zero. Ben-Tal and Nemirovski (1998, 1999, 2000); El-Ghaoui and Lebret (1997) and El-Ghaoui et al. (1998) propose less conservative models by considering uncertain linear problems with ellipsoidal uncertainty set independently. The ellipsoidal uncertainty set includes the "most likely" values of the uncertain parameters, and the robust counterpart of the nominal problem belongs to quadratic conic programming. Bertsimas and Sim (2004) propose a new robust approach that preserves the linearity and adjusts the level of conservatism of the problem flexibly. Bertsimas et al. (2004) introduce a D-norm that recovers the linear programming formulation of Bertsimas and Sim (2004). Bertsimas and Sim (2004) and Ben-Tal and Nemirovski (2000) assume that the random variables are independently and symmetrically distributed when obtaining probability bounds against constraint violation. However, in practice, the distributions are often known to be asymmetric. Many empirical studies show that portfolio returns are generally not normally distributed. Instead, their distributions are strongly asymmetric and fat-tailed (Fama, 1976; Kraus and Litzenberger, 1976; Kane, 1982; Duffee, 2002). In addition, the returns of portfolios involving credit derivatives can have extremely left-skewed distributions (Schönbucher, 2000; Lucas et al., 2001; Saunders et al., 2007). For this purpose Chen et al. (2007) introduce an uncertainty set using the forward and backward deviation measures of bounded random variables to capture the distributional asymmetry.

Natarajan et al. (2008) approximate the VaR of a portfolio based on robust optimization techniques. They optimize the asymmetry-robust VaR, which considers the asymmetry of the distribution of returns and is coherent. Moon and Yao (2011) propose a linear robust MAD model. They test the robust strategies on the real market data and study the performance of the robust model based on financial elasticity, standard deviation, and market condition such as growth, steady state, and decline in trend. Gülpınar et al. (2014) study the portfolio management problem with commodities and stocks under normal and disruptive conditions in petroleum markets. The robust investment strategies for different symmetric and asymmetric uncertainty sets are analyzed. In this paper, we build on their idea by suggesting an asymmetric robust MAD model based on the results of Chen et al. (2007).

Our contributions can be summarized as follows: First, we construct an asymmetry robust mean absolute deviation (ARMAD) model that captures the asymmetry distribution of the returns. Moreover, the ARMAD model can be formulated as a second-order cone programming problem (SOCP), which can be solved efficiently using the state-of-the-art SOCP solvers. Second, we study the in-sample and out-of-sample analysis of the robust strategies and compare the performance of the robust models with respect to the Chinese growing and declining market conditions. The experimental results show that the proposed asymmetry robust approach is able to distinguish the high return stocks. Hendricks et al. (1993); Carhart (1997) show that the relative performance of mutual funds persists in the near term. Portfolios of recent poor performers do worse than standard benchmarks, funds with high returns have higher-than-average expected returns in the near future. Therefore, the portfolios constructed by the ARMAD model can provide investors with good advice.

The remainder is organized as follows. Section 2 explains the mean absolute deviation (MAD) formulation. Section 3 contains a brief introduction to the robust portfolio management and the simple linear robust MAD model of Moon and Yao (2011). We develop an asymmetry-robust MAD problem for a factor model of the returns in Section 4. Section 5 focuses on the empirical analysis of the robust MAD models using Chinese real market data and presents computational results. Section 6 gives the conclusion.

Notations. Vectors are in lower case and matrices are in upper case. Tilde ($\tilde{\boldsymbol{z}}$) denotes uncertain parameters. ($\boldsymbol{z}'$) and (A') denote vector transpose and matrix transpose, respectively. $Diag(\boldsymbol{x})$ is a diagonal matrix whose diagonal entries are the elements of the vector $\boldsymbol{x}$.

2. Mean absolute deviation model

We will use the following notations throughout the paper:

n: the number of assets.
T: the time horizon.
M_0: the initial wealth of the investor.
x_j: the percentage of total wealth invested in asset j.
r_j: the expected return of asset j.
ρ: the minimal rate of return required by an investor.
l_j: lower bound on the weights of asset j.
u_j: upper bound on the weights of asset j.
r_{jt}: the realization of random variable R_j during period t.

Konno and Yamazaki (1991) propose the mean absolute deviation model, which is a linear programming and does not require calculation of the covariance matrix among assets. They show that the optimal portfolios and performance of MAD and Markowitz's mean-variance model are quite similar. Therefore, MAD can be used as an alternative to Markowitz's mean-variance model. The MAD model is:

$$\min_{x} \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{j=1}^{n}(r_{jt}-r_j)x_j\right|$$