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Portfolio selection with conservative short-selling

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ABSTRACT

Mean-variance analysis is considered the foundation of portfolio selection. Among various attempts to address the limitations of the original model as formulated by Markowitz more than 60 years ago, one simple solution has been to impose constraints on weights in order to reduce efficient portfolios with extreme weights that may be caused by estimation errors in the inputs. Although no short-selling constraints are often considered, the restriction removes opportunities to gain from short-selling and short positions provide various investment opportunities such as long/short strategies. In this paper we propose a portfolio selection model that allows short positions while examining the worst case only for assets that are assigned negative weights. The proposed model constructs portfolios with conservative short positions and the conservative level can be adjusted by the investor.

1. Introduction

Mean-variance analysis proposed by Markowitz (1952, 1959) is still considered the foundation of portfolio management and in recent years this framework for portfolio construction has become even more popular as a result of the advancement in automated investment management. When the mean-variance model is applied to asset allocation, the non-negativity constraint on portfolio weights is reasonable since the objective is often to divide the overall investment amount among several asset classes. But short positions in candidate assets can be beneficial and even necessary in portfolio selection when focusing on the construction of portfolios within an individual asset class.

Fundamentally, short sales allow investors to gain from assets that are overvalued. When short sales are not restricted, long/short strategies provide opportunities to exploit both long and short positions and the advantage of such strategies over long-only strategies has been widely studied (Jacobs and Levy, 1993, and Grinold and Kahn, 2000). For example, combining long and short positions allows constructing market-neutral strategies. It is also argued that restricting short sales leads to a less efficient market and higher volatility (Saffi and Sigurdsson, 2011, and Yeh and Chen, 2014). These observations illustrate the need for short sales in financial markets. More importantly, in portfolio construction, Levy (1983), Green and Hollifield (1992), and Brennan and Lo (2010) observe that mean-variance efficient portfolios contain negative weights. Furthermore, Behr et al. (2013) show that minimum-variance portfolios without constraints have lower variance and a higher Sharpe ratio than the portfolios constructed with short-sale constraints when the number of candidate assets is small. Finally, short positions in some assets allow taking larger long positions in other assets and thus become advantageous when large

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exposure is favored. In summary, short selling is an inevitable part of investing and the advantages motivate investors to consider short positions in their portfolios.

Nonetheless, short positions are generally associated with risky investment behavior. In fact, Jagannathan and Ma (2003) find that imposing no-shorting constraints may help reduce estimation errors in practice. However, imposing no-short-sale constraints is not always a recommended approach because many asset weights are bounded by the constraint and given zero weight (Black and Litterman, 1992). This is unappealing to investors because the recommended portfolio only invests in a few assets among many that are available. Thus, while no-shorting constraints prevent extreme weights that may appear in unconstrained portfolios, applying the constraint may be too restrictive because some assets in no-shorting portfolios may be given zero weight and thus being determined by the lower bound set by the investor.

In this paper, we propose a portfolio selection model based on the mean-variance framework that forms portfolios allocating short positions in a more conservative fashion. The model considers estimation errors in expected returns when finding the optimal short positions. By employing the proposed model, investors will be able to form portfolios that contain negative positions that are less risky than portfolios without constraints and yet more aggressive than portfolios with no-shorting constraints.

The remainder of the paper is organized as follows. Section 2 summarizes the mean-variance portfolio selection framework. Our proposed approach for forming portfolio with short sales is derived in Section 3. Simulation results comparing various portfolios are presented in Section 4, and Section 5 concludes the paper.

2. Mean-variance portfolio selection

The efficient portfolios in the mean-variance framework can be found using several formulations. The following formulation finds the portfolio with minimum variance among portfolios with expected return of at least r and bounds on weights imposed,

$$\begin{aligned} &\min_{\omega} \ \tfrac{1}{2}\omega^T\Sigma\omega \\ &\text{s.t.} \ \ \mu^T\omega \ge r,\ \omega^T\iota = 1,\ l \le \omega \le h \end{aligned} \tag{1}$$

where $\omega \in \mathbb{R}^n$ is the portfolio weight allocated to n assets, $\mu \in \mathbb{R}^n$ is the expected return of assets, $\Sigma \in \mathbb{R}^{n\times n}$ is the covariance matrix of asset returns, $r \in \mathbb{R}$ is the desired level of portfolio return, $\iota \in \mathbb{R}^n$ is the vector of ones, $l \in \mathbb{R}^n$ sets the lower bound on portfolio weights, and $h \in \mathbb{R}^n$ sets the upper bound on portfolio weights. We assume that $l \le 0$ and $0 \le h$ for the problems we are investigating.

Unconstrained portfolios are formed if the last constraint in (1) is removed, and short-sale-restricted portfolios are constructed if l is set to zeros and h is set to ones (or when h is dismissed). For portfolios with short-selling allowed, the lower bound l will contain negative values. We should note that the problem given by (1) may be infeasible if a high expected return is required while setting a tight bound on portfolio weights.

The major concerns with the basic mean-variance formulation given by (1) are the difficulty in accurately estimating the input values and the high sensitivity to changes in the input parameters (Best and Grauer, 1991). Therefore, large exposure, especially in short-selling, can be damaging when realized asset returns are different from the *ex-ante* estimates. We next present a revised mean-variance model that invests in short-sale positions cautiously where the conservative level can be adjusted.

3. The conservative short sale model

The proposed portfolio selection model is based on the classical formulation given by (1). The main idea behind our model is to consider the worst case of asset combination given negative weights for finding the optimal portfolio instead of restricting short positions altogether.

For considering the worst asset returns, we borrow the concept of uncertainty sets from robust optimization (see, for example, Fabozzi, Huang, and Zhou, 2010; Kim, Kim, and Fabozzi, 2014, 2016). If the expected return of an asset i is assumed to deviate at most δ_i from an estimated value $\hat{\mu}_i$, the uncertainty set for expected asset returns can be written as

$$\{\mu \mid |\mu_i - \hat{\mu}_i| \le \delta_i, i = 1, \ldots, n\} \tag{2}$$

where $\hat{\mu} \in \mathbb{R}^n$ is an estimate of the means and $\delta \in \mathbb{R}^n$ sets the possible deviation from the estimated value for each asset (Fabozzi et al., 2007). Hence, the worst expected return from investing in asset i when $\omega_i < 0$ is $\hat{\mu}_i + \delta_i$ because $(\hat{\mu}_i + \delta_i)\omega_i$ will have the smallest value among the possible outcomes from the uncertainty set. These observations lead to a formulation for forming portfolios with conservative short-selling, which is explained in the following proposition.

Proposition 1. *When the expected asset returns are described by the set given by (2), the following portfolio formulation finds the optimal portfolio with weights between l and h by considering the worst outcome for only the amount of short positions that exceed* γ,

$$\begin{aligned} &\min_{\omega_+,\omega_-} \ \tfrac{1}{2}\omega^T\Sigma\omega \\ &\text{s.t.}\ \hat{\mu}^T\omega - \delta^T\omega_- \ge r,\ \omega^T\iota = 1,\ \omega = \omega_+ - \omega_- - \gamma,\ 0 \le \omega_- \le -l - \gamma,\ 0 \le \omega_+ \le h + \gamma \end{aligned} \tag{3}$$