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Fama–MacBeth two-pass regressions: Improving risk premia estimates

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ABSTRACT

In this paper, we provide the asymptotic theory for the widely used Fama and MacBeth (1973) two-pass risk premia estimates in the usual case of a large number of assets. We demonstrate analytically and using simulations that the standard OLS and GLS estimators can contain large bias when the time series sample size is small, but our proposed OLS and GLS estimators can reduce the bias significantly.

1. Introduction

A fundamental problem in finance is to explain cross-sectional differences in asset expected returns. The CAPM of Sharpe (1964), Lintner (1965), and Black (1972) and various factor models as well as intertemporal/consumption models show that expected returns should be a linear function of asset betas with respect to economic fundamentals. There is a large literature that examines this linear asset pricing relationship (see, e.g., Jagannathan et al., 2010, for a survey). One of the most

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widely used econometric methodologies is the Fama and MacBeth (1973) two-pass regression. This procedure is used not only in asset pricing, but also in many other areas of finance and accounting.[1] Shanken (1992) provides an early study on its econometric properties. Recently, Shanken and Zhou (2007), Kan et al. (2013), among others, provide further analytical results and simulation evidence. However, except for Shanken (1992), almost all of the existing studies have focused on the case when the times series sample size, T, is larger than the number of assets, N. But in practice, most of the applications occur in the case of $N > T$.

In this paper, we provide the asymptotic theory and simulation evidence for the widely used FM OLS (Fama–MacBeth ordinary least squares) estimator when $N > T$. We show that the convergence of the FM OLS estimator depends crucially on T. This is contrary to the widely held belief that the larger the N, the greater accuracy the risk premium estimator. We demonstrate that this is not true. First, there is an order of $1/T$ bias for the estimator. Second, the asymptotic variance has two components, one of which is of order $1/T$ and the other is of $1/(NT)$. As a result, even if N is large, the first term will dominate the estimation error, and this term remains important even when N goes to infinity.

Because of the relatively small sample sizes used in empirical studies, we propose new OLS and GLS risk premia estimators that account for the bias caused by small T. As expected, the new estimators perform well in simulations for common sample sizes of $T = 60$ and $T = 120$. The reason that many or most cross-section regressions are run at these levels of relatively small T is the concern of parameter stability over time.

This article is organized as follows. In Section 2, we review the FM regression, and then, in Section 3, we present the asymptotic theory for the OLS estimator. In Section 4, we study the GLS estimator. In Section 5, we provide simulation evidence on the usefulness of the bias adjusted estimators. In Section 6, we conclude.

2. Fama–MacBeth regressions

In this section, we review the standard FM regression and the associated OLS estimator.

We assume that asset returns are governed by a multi-factor model:

$$R_{it} = \alpha_i + \beta_{i1} f_{1t} + \cdots + \beta_{iK} f_{Kt} + \epsilon_{it}, \quad i = 1, \ldots, N, \quad t = 1, \ldots, T, \tag{1}$$

where

R_{it} = the return on asset i in period t $(1 \leqslant i \leqslant N)$,

f_{jt} = the realization of the j-th factor in period t $(1 \leqslant j \leqslant K)$,

ϵ_{it} = the disturbances or random errors,

N = the number of assets, and T is the number of time-series observations.

In vector and matrix notation, we write the above model as

$$R_t = \alpha + \beta_1 f_{1t} + \cdots \beta_K f_{kt} + \epsilon_t = \alpha + B f_t + \epsilon_t,$$

where $R_t = (R_{1t}, \ldots, R_{Nt})'$ is the N-vector of asset returns; $\beta_1, \ldots, \beta_K$ are N-vectors of the multiple-regression betas; $B = (\beta_1, \beta_2, \ldots, \beta_K)$ is an $N \times K$ matrix; $f_t = (f_{1t}, \ldots, f_{Kt})'$, and $\alpha = (\alpha_1, \ldots, \alpha_N)'$. Let $\epsilon = (\epsilon_1, \ldots, \epsilon_T)$ be the $N \times T$ matrix of errors.

Like most studies, we maintain the standard assumption that the disturbances are independent of the common factors and that the disturbances are independent and identically distributed (iid) over time with mean zero and a nonsingular residual covariance matrix $\Sigma = E(\epsilon_t \epsilon_t')$, although this assumption can be relaxed with more complex formulas. The iid assumption is consistent with the behavior of stock returns data which have little correlations, and is also consistent with the underlying theoretical factor models which often are one-period models. Pedagogically, the iid assumption simplifies both notations and discussions, making it easy for readers at large to understand and apply the results. In what follows, we denote $E(\epsilon_{it}^2) = \sigma_{ii}^2$, and we also allow $E(\epsilon_{it}\epsilon_{jt}) \neq 0$, but the cross-sectional correlation must be weak so that $(NT)^{-1/2} \sum_{i=1}^{N} \sum_{t=1}^{T} \epsilon_{it} = O_p(1)$, i.e., the residuals are stochastically bounded.

[1] Applications of the procedure in recent years can be found in well over 9,690 papers that cite Fama and MacBeth (1973), as complied by Google Scholar.