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Quadratic hedging strategies for volatility swaps

Xingchun Wang [a], Jianping Fu [b], Guanying Wang [c,*], Yongjin Wang [d]

[a] School of International Trade and Economics, University of International Business and Economics, Beijing 100029, China
[b] College of Science, Tianjin University of Technology, Tianjin 300191, China
[c] College of Management and Economics, Tianjin University, Tianjin 300072, China
[d] School of Business, Nankai University, Tianjin 300071, China

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A B S T R A C T

This paper investigates a variance-optimal hedging strategy for volatility swaps under exponential Lévy dynamics. To obtain the optimal initial capital and the optimal amount of the underlying asset, we derive the explicit expressions of the Föllmer–Schweizer decomposition, which in turn implies the explicit expressions of hedging strategies. Numerical results are presented to show the performances of variance-optimal hedging strategies through comparing with other hedging methods.

1. Introduction

Volatility is one of the major features used to measure the fluctuations of asset prices. To manage the risk of the shifts in the volatilities of various assets, related financial instruments have been introduced. Among the most popular trading products, variance swaps and volatility swaps have been traded substantially in the markets to reduce the risk exposure to volatility risk. The payment of volatility swaps or variance swaps is path-dependent, which is defined by the realized volatility or variance of some underlying asset. Because of the square root relationship between volatility and variance, the results about

* Corresponding author. Tel.: +86 13920295882.
E-mail address: wangguanyingnk@163.com (G. Wang).

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pricing and hedging variance swaps do not hold for volatility swaps (Carr and Madan, 1998; Zhu and Lian, 2011). The evaluation of volatility swaps has been investigated recently (Jarrow et al., 2013; Lian et al., 2014). Unfortunately, it is difficult to replicate volatility swaps by virtue of pricing formula.

There are two major hedging approaches, superhedging strategy and maximizing expected utility. As pointed out by El Karoui and Quenez (1995), in complete markets, investors may gain profits but no losses by adopting superhedging strategy. However, only trivial superhedging strategy (i.e., buying and holding the hedging portfolios till the maturity without changes) exists for a large class of market models (Frey and Sin, 1999). Alternatively, investors could maximize some expected utility among all portfolios which have a fixed position in the contingent claim and different underlying assets (Cvitanić et al., 1999). Moreover, minimizing quadratic risk can be seemed as a special case of the second approach if we consider quadratic utility functions. Most recent literature on hedging volatility risk focuses on the second approach. Brenner et al. (2006) introduce a new volatility instrument which is termed as an option on a straddle. Broadie and Jain (2008) use variance swaps and a finite number of European call and put options to dynamically hedge volatility derivatives. Heston and Nandi (2000) propose a standard delta-hedging approach. When the dynamics of the underlying asset allows for jumps, delta-hedging strategy performs poorly because the underlying asset is not flexible enough to hedge the volatility. To address this issue, Windcliff et al. (2006) consider hedging strategies for discretely sampled volatility derivatives using a delta-gamma hedging strategy accompanied by appropriate vanilla hedging instruments. However, a natural question arises: how can we hedge volatility derivatives when other hedging instruments are not available? Here we consider a variance-optimal hedge strategy for volatility swaps, using the underlying asset only. This approach can be commonly-used to hedge volatility derivatives, even when other hedging products are not available in the markets. Windcliff et al. (2006) also show that the minimum variance optimal (MVO) hedge offers very little risk improvement over the unhedged position. Actually, the MVO hedge constructs a generalized trading strategy which is not necessarily self-financing and minimizes daily squared hedging errors. In addition, the local minimization of squared hedging errors could not guarantee the minimization of the global hedging risk, which is the target of variance-optimal hedging strategies considered in this paper. Therefore, the variance-optimal hedging strategy is a good candidate to hedge volatility derivatives, when there are no other traded derivatives on the underlying asset of volatility swaps.

In this paper, we consider a variance-optimal hedging strategy for volatility swaps under exponential Lévy dynamics. The Föllmer–Schweizer decomposition is used to obtain quadratic hedging strategies for volatility swaps. Although the decomposition is investigated in Schweizer (1994, 1995), they do not yield closed-form solutions. Adopting a new approach, we derive an explicit expression for a Föllmer–Schweizer decomposition in discrete time, which in turn implies explicit hedging strategies. Finally, we perform numerical analysis to illustrate hedging effects.

2. Hedging strategies in discrete time

In this section, we assume that the value of the underlying asset is described by a discrete-time version of exponential Lévy processes and derive the explicit expressions of hedging strategies. Let $(\Omega, \mathcal{F}, (\mathcal{F}_n)_{n\in\{0,1,\dots,N\}}, P)$ denote a filtered probability space. Denote by S the underlying asset price and let H be the payoff of a volatility swap, which has the following form,

$$H = \sqrt{\sum_{i=0}^{N-1}\left(\log\frac{S_{i+1}}{S_i}\right)^2}. \tag{2.1}$$

We replicate the payoff of volatility swaps approximately in variance-optimal sense. In the following, we aim to minimize the distance between the payoff of volatility swaps and the terminal value of the hedging portfolio, i.e., minimize the squared L^2-distance,

$$\mathbf{E}((c + G_N(\theta) - H)^2), \tag{2.2}$$

over all initial capital c and all admissible trading strategies $\theta \in \Theta$ where $G_N(\theta) = \sum_{i=1}^{N}\theta_i\Delta S_i$ denotes the cumulative gains from trade and Θ is the set of all predictable processes θ such that $G_n(\theta)$ is square-integrable for $n = 1, \dots, N$. Here we focus on hedging strategies by employing a quadratic criterion over