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Equilibrium option pricing: A Monte Carlo approach☆

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ABSTRACT

This paper presents a novel Monte Carlo method for option pricing that is based on a general equilibrium model. The advantage of the method compared to the standard risk-neutral pricing approach is that it does not require the specification of a market price of risk, making the method particularly suitable for pricing in incomplete markets. The method produces a strongly consistent estimator for the option price which exhibits the same error convergence rate as the standard risk-neutral pricing Monte Carlo approach. For illustration, the procedure is applied to the pricing of options under stochastic volatility.

Option valuation models are important in the theory of finance since many corporate liabilities can be interpreted as options or combinations of options. Using the assumption of no-arbitrage, financial economists have shown that the price of an option can be expressed in terms of the expected value of its discounted payout, where the expectation is taken with respect to a transformation of the original probability measure known as the *risk-neutral probability measure*. Unless the underlying market is complete, this transformation requires the specification of a market price of risk of the non-traded factors. This market price of risk is an unobservable variable with a value that changes constantly and whose specification typically requires significant assumptions. Bollen (1997) shows that different specifications of the market

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price of risk can lead to very different option prices and that an incorrect specification can have substantial effects for derivative valuation. Some researchers have avoided the problem of risk misspecification. For example, Jarrow and Madan (1997) and Husmann and Todorova (2011) present pricing models for options that are based on equilibrium considerations. However, their models rely on the standard CAPM, which only holds if either (i) asset returns are normally distributed or (ii) investors have quadratic utility functions.

The purpose of this paper is to present a novel Monte Carlo method for the pricing of options based on the general equilibrium model of Rubinstein (1976). The main advantage of this method compared to the standard risk-neutral pricing approach is that it does not require the specification of a market price of risk of the underlying factors. Thus, the method is particularly suitable in incomplete market settings where pricing by arbitrage considerations alone is precluded. The general equilibrium model developed by Rubinstein (1976) assumes an investor with CRRA power utility and does not place any restriction on the distribution of returns. Consequently, the method also avoids the restrictive assumptions underlying the standard CAPM used in previous studies. It is shown that this approach produces a strongly consistent estimator for the option price that can be expressed in terms of the ratio of two expectations and which has the same error convergence rate as the standard risk-neutral pricing Monte Carlo approach. Similar to the standard method, the efficiency of the estimation can further be improved by using variance reduction techniques or Quasi-Monte Carlo methods. Another shared feature with the standard Monte Carlo method is that the error convergence rate of the method is independent of the dimension of the problem.

The method is applied to the pricing of European options under the stochastic volatility model of Heston (1993), which is an important example of option pricing in an incomplete market setting due to the fact that volatility does not represent a traded asset. The simulation results underline the consistency of the developed method and highlight that the estimation error is inversely proportional to the number of simulation iterations n. The numerical analysis also explores the effects of stochastic volatility on equilibrium option prices and analyses the impact of the coefficient of relative risk aversion on option prices in equilibrium.

The rest of the paper is organized as follows. Section 1 introduces the general equilibrium model of Rubinstein (1976). Section 2 develops the Monte Carlo method and discusses its statistical properties. Section 3 applies the method to the pricing of options under stochastic volatility. Finally, Section 4 concludes.

1. The Rubinstein (1976) model

Rubinstein (1976) assumes that markets are frictionless, and that an investor is infinitely lived and chooses lifetime consumption and investment plans, subject to budget constraints, to maximize lifetime expected utility U, as represented by the time-separable power utility function:

$$U = \sum_{t=0}^{\infty} \delta^t E\left[\frac{C_t^{1-b}}{1-b}\right], \tag{1.1}$$

where C_t denotes consumption at time t, $\delta > 0$ is a measure of the investor's time preference, and $b > 0$ is the investor's constant coefficient of relative risk aversion.

Under these assumptions, Rubinstein (1976) shows that the present value PV_0 of any security that generates an uncertain future cash flow stream X_t can be found by:[1]

$$PV_0 = \sum_{t=1}^{\infty} \frac{E[X_t] - \lambda_t Corr[X_t, -(1+r_{m,t})^{-b}] Std[X_t]}{1 + r_{f,t}}, \tag{1.2}$$

where $r_{f,\,t}$ and $r_{m,\,t}$ are, respectively, the returns of an riskless asset and of the market portfolio over the time interval $(0, t]$; $Corr[x, y]$ is the correlation of x and y; $Std[\,\cdot\,]$ is standard deviation; and $\lambda_t = Std[(1 + r_{m,t})^{-b}]/E[(1 + r_{m,t})^{-b}]$.

[1] Note that there is a misprint in Eq. 3 in Rubinstein (1976). The numerator in Eq. 3 contains a covariance that should be a correlation. Eq. 2, from which Eq. 3 is derived, shows the correct term.

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