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Portfolio selection with independent component analysis

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We analyze a methodology for portfolio selection based on the independent component analysis. In this paper parametric and non-parametric approaches are used for capturing the behavior of independent components that generate the distribution of asset returns. Although the setup is quite general, we focus mainly on the numerical issues encountered for parametric models and suggest the inclusion of a penalty function in the optimization problem.

1. Introduction

Different authors (Jondeau and Rockinger, 2006; Martellini and Ziemann, 2010) have remarked that investor's preferences go beyond the decision based on the first two moments of the return distribution. The models proposed in literature that incorporate higher moments in portfolio selection are divided into non-parametric (see, among others, Hitaj et al., 2012, and the literature therein) and parametric (see Jondeau et al., 2007). The advantage of using parametric approaches is the reduction of the number of parameters to estimate if we compare them with non-parametric models (see Hitaj and Mercuri, 2013; Jondeau et al., 2007, and references therein). However, when dealing with portfolio returns, we need to tackle with multivariate distributions.

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The idea of this paper is to model log returns in a portfolio adopting the perspective of independent components analysis (ICA) (see Hyvarinen et al., 2001), where the independent components are fitted using univariate infinitely divisible distributions. A multivariate Lévy model is proposed in Ballotta and Bonfiglioli (2014), and its estimation is considered in Loregian et al. (2015), but in practical implementations it considers one common factor only, while the approach here pursued easily cope with a multiplicity of independent factors.

In the ICA approach the dependence structure of the assets in the portfolio is described through the mixing matrix that is an output of the `FastICA` algorithm proposed in Hyvarinen (1999) based on a two-step prewhitening procedure that potentially can reduce the number of components to consider. In fact, if the covariance matrix of the sphered data is singular or near-singular the number of components should be lower than the number of the observed variables. Each distribution is then fitted to the independent components while portfolio returns are reconstructed on their linear transformation. From the independence assumption for the components, we can express the expected utility as a product of some terms that depend on the distribution used for modeling the components.

First we discuss two non-parametric methods respectively historical and exponential decay. The historical distribution is the most used in finance since it is simple to deal with. The exponential decay is similar but is based on the assumption that recent data are more reliable for predicting future movements of asset prices. In this paper we consider also two parametric distributions for modeling the components: the Variance Gamma and the Mixed Tempered Stable. Both these distributions account for higher moments and present nice mathematical tractability. The Variance Gamma distribution has been widely applied in finance (see among others Madan and Seneta, 1990; Carr and Madan, 1999; Fiorani, 2004) while the Mixed Tempered Stable (MixedTS) is a new distribution introduced in Rroji and Mercuri (2014). This latter distribution can be seen as a generalization of the Normal Variance Mean Mixtures (NVMM see Barndorff-Nielsen et al., 1982) since it has a similar structure and its definition generates a dependence of higher moments on the parameters of the Tempered Stable (see Küchler and Tappe, 2013, recently) that replaces the Normal distribution. In a model based on the Mixed Tempered Stable it is possible to deal with fat tailed portfolio returns because the Tempered Stable has the α-stable distribution as a limit case while in NVMM we need an ex ante decision on the mixing random variable.

In the empirical analysis we perform a comparison in terms of performances of the portfolios obtained maximizing the expected utility of a Constant Absolute Risk Aversion (CARA) utility function (see for example Nocetti, 2006) for the four distributions used to model the components with the performances of the Energy Select Sector SPDR fund. We perform a rolling analysis and investigate not only absolute variations of the invested sum but also the stability of the results. Portfolio diversification results for both parametric and non-parametric approaches are displayed by means of the normalized Herfindahl index.

The outline of the paper is as follows. In Section 2 we describe the optimization problem based on the ICA analysis. In Section 3 we use the Variance Gamma and generalize the procedure explained in Madan and Yen (2004) in the sense that dimension reduction is possible while in Section 4 the Mixed Tempered Stable is used for modeling the time series of the independent components. The methodology is employed in Section 5 for an empirical analysis. Section 6 concludes the paper.

2. Portfolio selection with ICA

We consider a portfolio composed by $\bar{n}$ assets whose log returns are described through a linear transformation of independent signals:

$$r = AS \tag{1}$$

where $r_{\bar{n}\times t} = [r_1, \ldots, r_{\bar{n}}]$ is a matrix that contains the ($\bar{n}$) asset returns placed in each column. A is a $\bar{n} \times p$ mixing matrix obtained applying the `FastICA` algorithm and S the $p \times t$ matrix of signals. It is worth noting that $\bar{n} \geq p$ since we could decide to perform a Principal Component Analysis in order to reduce the number of signals before extracting the ICs. The ICA algorithm yields a linear transformation of observable signals r using the matrix W, that is an estimate of the inverse of matrix A, such that the obtained signals $\hat{S} = Wr$ are as independent as possible. $\hat{S}$ contains the estimates of the underlying independent components in S. The problem is well-defined if and only if the components S_i for $i = 1, \ldots, p$ are non-gaussian and these components are uniquely retrieved up to a permutation. There are different statistical