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Enhanced index tracking optimal portfolio selection

Wanderlei Lima de Paulo[a,*], Estela Mara de Oliveira[b], Oswaldo Luiz do Valle Costa[b]

[a] *College of Economics, Business and Accounting, University of Sao Paulo, Av. Prof. Luciano Gualberto, 908, São Paulo (SP) 05508-900, Brazil*
[b] *Department of Telecommunications Engineering and Control, University of Sao Paulo, Av. Prof. Luciano Gualberto, 158, São Paulo (SP) 05508-900, Brazil*

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ABSTRACT

In this paper we present an analytical solution for an uni-period enhanced index tracking problem with limited number of assets held in the tracking portfolio. We consider an approach in which the tracking portfolio is composed of a given subset of assets, and the value function is written as the trade-off between the tracking error and excess return, balanced by an appropriate choice of a risk aversion parameter. This formulation allows an analytical comparison of the betas and value functions of the optimal portfolios with and without tracking. Our approach provides readily implementable formulae, being consequently easier for numerical implementation.

1. Introduction

Since the classical mean-variance model proposed by Markowitz (Markowitz, 1952), the asset allocation problem has been widely studied in the finance literature. More recently the portfolio optimization problem has been extended to cover several different situations such as robust portfolio optimization, multi-period mean-variance models and portfolio selection problems in the presence of regime switching (see, for example, Costa and Paiva, 2002; Li and Ng, 2000; Kim et al., 2013 and Bae et al., 2014).

* Corresponding author. Tel.: +55 11 3019 5820.
E-mail addresses: wldepaulo@gmail.com (W.L. de Paulo), estelaime@hotmail.com (E.M. de Oliveira), oswaldo@lac.usp.br (O.L. do Valle Costa).

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Another type of portfolio optimization problem consists of establishing an optimal allocation so that the portfolio's return replicates the return of a market index (benchmark). This problem is the so-called index tracking problem, where the utility tracking error function is based on the difference between the portfolio's return and the benchmark's return (problems of this nature are addressed in Roll (1992); Alexander and Baptista (2010) and Bajeux-Besnainou et al. (2011), for example). In the same spirit, the problem known as enhanced index tracking (or enhanced indexation) aims at obtaining returns above the reference index (excess return), while minimizing the tracking error. This kind of problem is studied in Wu et al. (2007); Canakgoz and Beasley (2008); Li et al. (2011) and Guastaroba and Speranza (2012).

In general, the asset allocation problem aims at minimizing the portfolio variance under specified restrictions such as limitations on the weights of the assets, liquidity constraints and limitation on the number of assets included in the portfolio. Because of its practical relevance, the portfolio optimization model with limited number of assets has been intensively studied in the literature (see, for instance, Cesarone et al., 2013 and Fastrich et al., 2014). Particularly, this kind of problem is inherent in the passive portfolio management (tracking problem) where the goal is to find a portfolio with a small number of assets to track a chosen benchmark. There are many papers in the literature dealing with the problem of limiting the number of assets in the index tracking and enhanced index tracking problems (see Beasley et al., 2003; Canakgoz and Beasley, 2008 and Guastaroba and Speranza, 2012, for example). Usually it is imposed cardinality constraints to restrict the number of assets, so that it is possible to construct a tracking portfolio selecting an optimal subset of assets that are present in the benchmark index.

Another approach, as used in Yao et al. (2006) and Edirisinghe (2013), is to consider the tracking portfolio for a given subset of assets belonging to the market index. In Yao et al. (2006) this kind of problem is considered under a continuous-time infinite horizon discounted stochastic linear quadratic control problem framework while in Edirisinghe (2013) the author focus on the uni-period problem of minimizing the variance of the tracking error subject to a specified mean tracking error return, and provides an analytical solution for the problem as well as a comparison with the classical Markowitz mean-variance model.

Differently from Yao et al. (2006), which considered an infinite horizon continuous-time horizon discounted stochastic linear quadratic control problem, in this paper we focus on obtaining an analytical solution for an uni-period enhanced index tracking problem with limited number of assets held in the tracking portfolio. When compared with Edirisinghe (2013) we should point out that the goals of the papers are different since Edirisinghe (2013) aims at minimizing the variance of the tracking error subject to a specified mean tracking error return while the present paper aims at minimizing an utility tracking error function based on the difference between the portfolio's return and the benchmark's return. As it will be further explained in Section 2.1 the advantage of this formulation is that the trade-off between the tracking error and excess return can be balanced by an appropriate choice of a risk aversion parameter. Moreover, this kind of objective function naturally arises when considering the maximization of the expected value of a negative exponential utility function for the normal case. Furthermore, our formulation also allows, similarly as in Edirisinghe (2013), an analytical comparison between the optimal portfolios with and without tracking. Finally it is worth noting that our approach provides closed and readily implementable formulae, being consequently easier for numerical implementation than the semidefinite programming computational tool used in Yao et al. (2006).

As aforementioned, we compare the solution obtained for the enhanced index tracking problem with the one derived without the tracking requirement in the objective function and we find that (i) the tracking optimal portfolio beta increases with respect to the beta of the optimal portfolio without tracking; (ii) the value function of the problem without tracking for the tracking optimal portfolio increases with respect to the one for the optimal portfolio without tracking; (iii) the value function of the tracking problem for the tracking optimal portfolio decreases with respect to the one for the optimal portfolio without tracking. Furthermore, we can see that the performance of the considered enhanced indexation problem strongly depends on the set of assets selected as the tracking portfolio.

The remainder of the paper is organized as follows. Section 2 includes the formulation to the enhanced index tracking problem with limited number of assets held in the tracking portfolio, an analytical formula for the optimal tracking portfolio composition and a comparison between the tracking optimal portfolio and the optimal portfolio without tracking. In Section 3 a numerical example for an enhanced indexation portfolio using a few stocks is presented. Finally, Section 4 presents some final remarks.