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Option pricing on foreign exchange in a Markov-modulated, incomplete-market economy

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In this study, we investigate the currency option pricing in a Markov-modulated, incomplete-market economy. Specifically, the dynamics of the spot foreign exchange rate and the domestic/foreign instantaneous forward interest rates are, respectively, governed by a two-factor Markov-modulated stochastic volatility model with jumps and a Markov-modulated Heath–Jarrow–Morton model. The analytical expressions are obtainable using the random Esscher transform. Numerical examples are also given.

1. Introduction

A variety of foreign exchange-linked instruments have been invented for hedging the oscillating risk generated by the currency market. Biger and Hull (1983) and Garman and Kohlhagen (1983) derive the pricing formulas for an option on foreign exchange (FX), in which the diffusion dynamics of the spot FX rate are log-normal. Nevertheless, increasing evidences have indicated that the geometric Brownian motion is not completely consistent with the reality (Jorion, 1988; Bates, 1996). Ahn et al. (2007) study the valuation of currency options when the spot FX rate dynamics are governed by jump-diffusion models. In the actual FX market, the spot FX rate may change over time according to the state of an economy.

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To price currency options under switching regimes, Siu et al. (2008) propose a two-factor stochastic volatility model for pricing currency options. Bo et al. (2010) investigate the valuation of currency options under a Markov-modulated jump-diffusion process where the time-varying transition rates are modulated by a hidden Markov chain. Using the Markov-modulated Lévy dynamics, Swishchuk et al. (2014) generalize the pricing formulas in Bo et al. (2010) for the currency option prices. Unfortunately, these aforementioned pricing methods do not integrate a full term structure model of interest rates into the valuation framework. Amin and Jarrow (1991) give a general framework to price currency options under stochastic interest rates using the Heath et al. (1992) model by specifying the dynamics of instantaneous forward rates. Valchev (2004) extends the class of deterministic volatility Heath–Jarrow–Morton models to a Markov chain stochastic volatility framework. Under the Valchev (2004) model, Chen et al. (2014) investigate the no-arbitrage price of quanto options.

In this study, the results of Siu et al. (2008) and Bo et al. (2010) are generalized to a case when the spot FX rate dynamics and the forward interest rate processes are, respectively, driven by a two-factor model of Markov-modulated stochastic volatility with jumps (MMSVJ) and a Markov-modulated Heath–Jarrow–Morton (MMHJM) model. The model parameters are all modulated by a hidden Markov chain whose states represent the hidden states of an economy. Due to the Markov-modulated risks, the market is therefore incomplete. The risk brought by switching regimes can be hardly diversified since it is more likely to be regarded as a systematic risk. In this circumstance, the Esscher transform developed by Gerber and Shiu (1994) is one of the major tools for pricing options in an incomplete market. Elliott et al. (2005) propose the use of the Esscher transform for option valuation in a Markov-modulated economy and justify its use by the minimal entropy martingale measure. Under the Markov structure, growing studies (Siu et al., 2008; Bo et al., 2010; Swishchuk et al., 2014; Lian et al., 2015) employ the Esscher's change of measures for underlying asset price dynamics to determine a pricing kernel. We are interested in finding formula to solve such an option pricing problem in our model framework. In this study, we extend the Esscher transform technique to obtain analytical pricing formulas based on solving a system of Markov-modulated Esscher parameters.

The remainder of this study is organized as follows. The next section represents the dynamic model. Section 3 illustrates the measure change and the currency option pricing. Section 4 provides the numerical experiments. The final section shows the conclusions of this study.

2. Model framework

Let (Ω, F, P) be a complete probability space, where P is the physical measure. For all $t \in [0, T]$, we define a continuous-time, finite-state Markov chain $\xi(t)$ on (Ω, F, P) with state space $\Im = \{1, 2, \ldots, I\}$. The states of $\xi(t)$ can be interpreted as the hidden states of an economy. Without loss of generality, we can take the state space $\Im$ for $\xi(t)$ to be a finite set of unit vectors $(e_1, e_2, \ldots, e_I)$ with $e_j = (0, \ldots, 1, \ldots, 0) \in \mathbb{R}^I$. Elliott et al. (2005) give the semi-martingale representation for $\xi(t)$ as follows:

$$d\xi(t) = \Pi\xi(t)dt + dM(t), \tag{1}$$

where $M(t)$ is an $\mathbb{R}^I$-valued martingale with respect to the natural filtration generated by $\xi(t)$ under P. Let $\Pi = (\pi(i, j))_{i,j=1,2,\ldots,I}$ denote the generator or rate matrix of $\xi(t)$ under P, where $\pi(i, j)$ is the transition intensity of $\xi(t)$ from state e_i to state e_j.

Let $X(t)$ represent the domestic price at time t of one unit of the foreign currency. Here, we construct a two-factor MMSVJ model to describe the dynamic behavior of spot FX rates, which is given by the following:

$$\frac{dX(t)}{X(t-)} = (\mu(t) - \lambda(t)\kappa)dt + \sqrt{V(t)}dW_1(t) + \sigma(t)dW_2(t) + (\exp(Z(t-)) - 1)dN(t),$$

$$\frac{dV(t)}{V(t-)} = \alpha dt + \beta dW_v(t), \tag{2}$$

for all $t \in [0, T]$. In which, the first stochastic volatility $V(t)$ of the spot FX rate follows a log-normal diffusion. The appreciation rate $\mu(t)$ and the second stochastic volatility $\sigma(t)$ of the spot FX rate and the stochastic arrival intensity $\lambda(t)$ of the Poisson process $N(t)$ are all modulated by $\xi(t)$ as follows:

$$\mu(t) = \langle \mu, \xi(t) \rangle, \quad \mu = (\mu(1), \mu(2), \ldots, \mu(I)) \in \mathbb{R}^I, \tag{3}$$