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Pricing power exchange options with correlated jump risk

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ABSTRACT

This paper extends the framework of Blenman and Clark (2005) to value power exchange options by incorporating correlated jump risk. A typical class of jump-diffusion processes are used to describe the values of two risky assets, and a common jump process is introduced to allow for correlated jump risk. In this framework, I obtain an explicit pricing formula for power exchange options, and illustrate the effects of common jump risk as well as the difference between the impacts of idiosyncratic and common jump risk.

1. Introduction

In this study, I consider a valuation model for power exchange options with correlated jump risk. Power exchange options are a generalization of power options (see, e.g., Tompkins, 1999) as well as Fischer–Margrabe exchange options (see, e.g., Fischer, 1978; Margrabe, 1978). These options are rather powerful financial tools in the fields of hedging nonlinear risk or compensation designs. Margrabe (1978) studies the value of exchange options, which allow option holders to exchange one asset for another at the maturity. Fischer (1978) also investigates the pricing issue of exchange options and accounts for the situation where exercise price is the price of an untraded asset. Tompkins (1999) mainly focuses on the applications of power options in hedging nonlinear risk. Among the various applications of exchange options, it is worth mentioning executive compensation options. Generally speaking, exchange options can be seen as indexed executive stock options (ESO), which have been used widely to align the chief executive officers' (CEOs) interest with shareholders' as an important compensation scheme (see, e.g., Johnson and Tian, 2000; Armstrong and Vashishtha, 2012).

Blenman and Clark (2005) generalize exchange options and power options to power exchange options, and explicitly solve for the price of power exchange options, under the assumption that the underlying assets' values are governed by geometric Brownian motions. However, contrary to the assumption of continuous sample paths, the arrival of new information[1] may cause discontinuous changes in asset prices. To capture discontinuous asset prices, an increasing number of stochastic processes are employed to describe asset prices, including compound Poisson processes, exponential jump-diffusion processes, and general Lévy processes. Merton (1976) employs compound Poisson processes to model discontinuous changes of stock prices. In Madan et al. (1998), the variance gamma process is used to model the dynamics of log stock prices.

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[1] Evidence for jump risk has been illustrated in the literature, e.g., Bakshi et al. (1997); Aït-Sahalia (2002); Bakshi et al. (2003) and (Bates, 2008).

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Kou (2002) adopts the double exponential jump-diffusion processes and considers the valuation of European options. Cai and Kou (2011) investigate option prices under the mixed-exponential jump-diffusion processes.

The purpose of this paper is to provide a pricing model for power exchange options with correlated jump risk. By adopting jump-diffusion processes to describe the dynamics of asset prices, I incorporate discontinuous changes in both risky asset prices, which improves the framework of Blenman and Clark (2005). Further, I break down the total jump risk into idiosyncratic component and common jump risk, which allow me to consider the impact of correlation in a quite general framework and investigate the difference between the impacts of idiosyncratic and common jump risk. In addition, the correlation between two risky assets is incorporated in both the continuous part and the discontinuous part, and correlation for the discontinuous process part is linked by a common jump process. In this framework, I obtain an explicit formula for power exchange options, which encompasses many existing formula as special cases, including Black and Scholes (1973); Merton (1976) and Blenman and Clark (2005).

In the numerical part, I illustrate the impacts of correlated jump risk on power exchange option prices, as well as the impacts of correlation coefficient in the continuous part. I find that a lower correlation coefficient between the assets corresponds to a higher option price, which is consistent to our intuition. Opposite changes in the values of two assets with a negative correlation coefficient are more likely to happen, making options more valuable. Option prices increase with jump intensities corresponding to idiosyncratic jump risk. Here I am more interested in the effects of common jump risk as well as the difference between the impacts of idiosyncratic and common jump risk. Since the common jumps affect both assets at the same time, the effects are not intuitive. I find that an increase of common jump risk always corresponds to an increase of option prices, but increasing the proportion of common jump risk has a bigger impact when the total jump risk is unchanged.

The remainder of this paper is organized as follows. In Section 2, the proposed model is described and the detailed calculation for an explicit pricing formula is given. Section 3 is devoted to numerical results. Finally, Section 4 concludes the paper.

2. The theoretical framework

In this section, I deal with the pricing issue of power exchange options with correlated jump risk. Here I adopt jump-diffusion processes to describe the values of two risky assets S_1 and S_2, and introduce a common jump process to capture correlated jump risk. Meanwhile, I demonstrate in detail how to derive the final pricing formula.

Assume that the uncertainty of the economy is described by a probability space (Ω, F, P). The values of two risky assets are assumed to follow jump-diffusion processes,

$$\frac{\mathrm{d}S_i(t)}{S_i(t-)} = \mu_i \mathrm{d}t + \sigma_i \mathrm{d}W_i(t) + (e^{Z_{i,0}(t-)} - 1)\mathrm{d}N(t) + (e^{Z_{i,1}(t-)} - 1)\mathrm{d}N_i(t), \tag{2.1}$$

where for $i = 1, 2,\ \ \mu_i \in R,\ \sigma_i > 0$ denotes the volatility of the asset S_i and $W_1(t)$ and $W_2(t)$ are two standard Brownian motions with a correlation coefficient ρ, i.e., $\langle \mathrm{d}W_1(t), \mathrm{d}W_2(t)\rangle = \rho \mathrm{d}t$. $N(t)$, $N_1(t)$ and $N_2(t)$ are three independent Poisson processes with intensities λ, λ_1 and λ_2, which are used to model discontinuous changes of asset prices. Specifically, discontinuous changes in asset prices are divided into two parts: individual ones, corresponding to $N_i(t)$, and common ones, corresponding to $N(t)$. Here I take $N(t)$ as the common risk factor for two risky assets and I will investigate how $N(t)$ affects option prices in the numerical section.

If the common jump happens at time t, the jump amplitude of the asset $S_i,\ \ i = 1, 2,$ is controlled by $Z_{i,\,0}(t)$. For any time $t \neq s$, I assume that $Z_{i,\,0}(t)$ and $Z_{i,\,0}(s)$ are independently and identically distributed. Similarly, the individual jump amplitude of the asset $S_i,\ \ i = 1, 2,$ corresponding to $N_i(t)$, is assumed to be controlled by $Z_{i,\,1}(t)$.

For valuation purposes, this study follows Merton (1976) and assumes that the jump risk that is common to both assets is diversifiable in the market and has a zero risk premium. The assumption ensures that there exists a risk neutral measure Q. Under Q, the dynamics of two risky assets prices become

$$\frac{\mathrm{d}S_i(t)}{S_i(t-)} = (r - k_{i,0}\lambda - k_{i,1}\lambda_i)\mathrm{d}t + \sigma_i \mathrm{d}W_i(t) + (e^{Z_{i,0}(t-)} - 1)\mathrm{d}N(t) + (e^{Z_{i,1}(t-)} - 1)\mathrm{d}N_i(t), \tag{2.2}$$

where for $k_{i,j} := \mathrm{E}[e^{Z_{i,j}(t)}] - 1$ is the mean percentage jump of the price and is assumed to be finite for $i = 1, 2$ and $j = 0, 1$. Specifically, as in Merton (1976), $Z_{i,\,j}$ is normally distributed with mean $\mu_{i,\,j}$ and standard deviation $\sigma_{i,\,j} > 0$. Then, $k_{i,j} = e^{\mu_{i,j} + \frac{1}{2}\sigma_{i,j}^2} - 1$.

In the following, I derive the explicit formula for power exchange options with normally distributed jump amplitude $Z_{i,\,j}$. A power exchange option is a European option to exchange the power value $\beta_1 S_1^{\alpha_1}$ of one asset to the power value $\beta_2 S_2^{\alpha_2}$ of another asset. Denote by C^* the power exchange option price, which is represented by,

$$C^* = e^{-rT}\mathrm{E}\Big[\big(\beta_1 S_1^{\alpha_1}(T) - \beta_2 S_2^{\alpha_2}(T)\big)^+\Big].$$

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