Finance Research Letters 000 (2016) 1–7

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Dynamic consumption and portfolio choice with permanent learning

Hyun-Tak Lee

Department of Industrial and Management Engineering, POSTECH, 77 Cheongam-Ro, Nam-Gu, Pohang, 37673, Republic of Korea

ARTICLE INFO

Article history:
Received 11 May 2016
Revised 14 June 2016
Accepted 1 July 2016
Available online xxx

JEL classification:
C61
G11

Keywords:
Optimal consumption
Optimal portfolio
Information quality
Learning

ABSTRACT

This paper studies a continuous–time intertemporal consumption and portfolio choice problem when a long–horizon investor who has recursive preferences cannot exactly observe the expected returns of the risky asset. I contribute to belief–behavior solutions to the explicit log-utility case, and to the approximate unit-risk-aversion case. I show explicitly that her belief behavior depends on the parameters of investment opportunities and investor preferences.

1. Introduction

This paper studies how *information quality* affects intertemporal optimal consumption and portfolio choice. In financial economics, information quality related to *parameter learning* represents how updates to information affect asset pricing or investment decisions. Standard portfolio literature (Campbell et al., 2004; Chacko and Viceira, 2005) has commonly assumed that investors have complete knowledge of probability distributions (complete information), so several dynamics that influence their wealth have fixed parameters. However, in practice, these parameters are unknown (incomplete information), so investors must learn them from observed data for use in making forward–looking decisions.

To express this idea precisely, I develop a model in which a long–horizon investor with a recursive utility (Duffie and Epstein, 1992) cannot exactly observe the expected returns of a risky asset; simply, the expected–return process is assumed to follow a two–state and continuous hidden Markov chain. The representative investor learns the time–varying expected returns from risky asset prices by revising her beliefs.

Specifically, the representative investor has prior belief about the current regime of a financial market, and uses this belief to infer the unknown expected returns. I show that prior belief affects myopic portfolio demand; this short–term investment characteristic is the same for both single–period (myopic) and multi–period investors. At the same time, she continuously updates posterior beliefs with regard to future variation in expected returns. I show that revisions in posterior beliefs governed by a nonlinear mean–reverting stochastic differential equation (SDE) have a *permanent* effect on time variation in investment opportunities.[1] This permanent nature gives rise to *nonlinear* intertemporal hedging demand against

E-mail address: lht1107@postech.ac.kr

[1] A nonlinear SDE means that the SDE does not have an affine diffusion term with respect to a state variable. See (1).

http://dx.doi.org/10.1016/j.frl.2016.07.001
1544-6123/

adverse shifts in future investment opportunities; this conclusion differs from the linear hedging demand of the standard literature.

Given the rational expectations model, I explore how revisions in posterior beliefs affect optimal consumption and portfolio policies. In particular, I contribute to belief–behavior solutions to a log-utility case, and to a unit-risk-aversion case.

First, I obtain the explicit solution to the log–utility case: the elasticity of intertemporal substitution (EIS) of consumption and the coefficient of relative risk aversion (RRA) are both equal to one. The conventional wisdom is that a logarithmic investor does behave like a myopic investor. Even though the logarithmic investor focuses only on the short–term investment characteristic, I show that she also updates her posterior beliefs. Her belief behavior depends on the parameters of investment opportunities and investor preferences.

Second, I derive the approximate solution to the unit–risk–aversion case by further relaxing that EIS = 1. The interpretation is similar to the log–utility interpretation in terms of the updating behavior. By contrast, I show that the size of the EIS determines the relative importance of the substitution and income effects of belief change on consumption. The relative importance has a crucial difference on how posterior–belief variation affects the optimal consumption rule.

The rest of the paper is organized as follows. Section 2 states the investment opportunity set and optimization problem. Section 3 studies the optimal policies in response to posterior–belief variation and the resulting belief behavior with the two cases. Section 4 concludes.

2. The intertemporal consumption and portfolio selection

2.1. Investment opportunity set

Simply, the investor trades two investment assets. The first one with instantaneous return r is riskless:

$$\frac{dB(t)}{B(t)} = rdt.$$

The second one with time–varying expected return $\mu(t)$ is risky:

$$\frac{dS(t)}{S(t)} = \mu(t)dt + \sigma dZ(t),$$

where the expected return $\mu(t)$ is governed by the two–state hidden Markov chain: high regime H, and low regime L with $\mu_H > \mu_L$; $\sigma > 0$ is a constant standard deviation; and $Z(t)$ is a standard Brownian motion.

The investor does not exactly know the expected return $\mu(t)$ (nor the resulting Brownian motion). To infer the unknown $\mu(t)$, she uses own prior belief $p(t) \equiv \Pr(\mu(t) = \mu_H) \in [0, 1]$ at initial time t and then updates posterior beliefs with regard to future variation in expected returns. Given information available at time t, her estimated expected return $\bar{\mu}(t)$ is defined as

$$\bar{\mu}(t) \equiv \mu_H \cdot p(t) + \mu_L(1 - p(t)) = \mu_L + (\mu_H - \mu_L)p(t).$$

The use of a nonlinear filtering theory (Liptser and Shiryaev, 2001) delivers the filtered asset process as

$$\frac{dS(t)}{S(t)} = \bar{\mu}(t)dt + \sigma d\widehat{Z}(t),$$

and the nonlinear mean–reverting posterior–belief process as

$$\begin{aligned} dp(t) &= \{\lambda_L - (\lambda_H + \lambda_L)p(t)\}dt + \frac{\mu_H - \mu_L}{\sigma} p(t)(1 - p(t))d\widehat{Z}(t) \\ &\equiv \mu_p(p(t))dt + \sigma_p(p(t))d\widehat{Z}(t), \end{aligned} \tag{1}$$

where λ_i for $i, j \in \{H, L\}$ is a transition intensity such that regime i jumps to regime j for $j \notin i$, and the new filtered Brownian motion $\widehat{Z}(t)$ is defined as

$$\widehat{Z}(t) = \int_0^t \frac{dS(u) - \bar{\mu}(u)S(u)du}{\sigma S(u)}.$$

The resulting investment opportunity set describes her wealth process $W(t)$ evolving as

$$\begin{aligned} dW(t) &= \left[\{r + \pi(t)(\bar{\mu}(t) - r)\}W(t) - C(t)\right]dt + \sigma\pi(t)W(t)d\widehat{Z}(t) \\ &\equiv \mu_w(W(t), p(t), C(t), \pi(t))dt + \sigma_w(W(t), \pi(t))d\widehat{Z}(t), \end{aligned} \tag{2}$$

where $C(t)$ denotes consumption, and $\pi(t)$ is the proportion of wealth invested in the risky asset.

The investment opportunity set differs from the set used in Campbell et al. (2004). First, the expected–return process $d\bar{\mu}(t) = (\mu_H - \mu_L)dp(t)$ includes the concave diffusion term $(\mu_H - \mu_L)\sigma_p \equiv \frac{(\mu_H - \mu_L)^2}{\sigma} p(t)(1 - p(t))$ that depends on prior belief, whereas Campbell et al. (2004) consider the constant diffusion case. Second, I study a complete–market case, in which shocks to $d\bar{\mu}(t)$ are perfectly correlated with shocks to $dS(t)/S(t)$. By contrast, Campbell et al. (2004) study an incomplete–market case, in which shocks to $d\mu(t)$ are *not* perfectly correlated with shocks to $dS(t)/S(t)$ with a negative correlation