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Dating the financial cycle with uncertainty estimates: a wavelet proposition

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A B S T R A C T

We propose to date and analyze the financial cycle using the Maximum Overlap Discrete Wavelet Transform (MODWT). Our presentation points out limitations of the methods derived from the classical business cycle literature, while stressing their connection with wavelet analysis. The fundamental time-frequency uncertainty principle imposes replacing point estimates of turning points by interval estimates, which are themselves function of the scale of the analysis. We use financial time series from 19 OECD countries to illustrate the applicability of the tool.

1. Introduction

Since the 2008 financial crisis, understanding the stylized empirical regularities of the financial cycle has emerged as an important research question (Borio, 2014). Indeed, to comprehend the influence of financial upturns and downturns on macroeconomic cycles, we first need to identify these stylized facts.

Dating and analyzing financial cycles requires suitable tools. In this regard, scholars have borrowed substantially from the business cycle literature, mainly by employing the Bry-Boschan Quarterly (BBQ) algorithm for the detection of business cycle turning points, or one of its variations (Claessens et al., 2012). Similarly, studies interested in medium term fluctuations have used band-pass filters in order to isolate the periodic components of financial time series ranging from 8 to 16 years (Drehmann et al., 2012). Research has not, however, sufficiently explored the meaning of the outputs of the BBQ algorithm in the presence of the medium-term dynamics of financial time series. Most analyzes tend to either disregard the frequency properties of the cycles or to conduct time and frequency analysis independently. Failing to make an explicit connection between the time and frequency domains can cause great misunderstanding, as more than 50 years of research in business

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cycles has evidenced (Harding and Pagan, 2005). Moreover, connecting time and frequency analyzes is especially relevant in the current state of the literature, when we have yet to understand the financial cycle and its properties.

Building upon the insights of Yogo (2008), Michis (2014), and Lera and Sornette (2015) who discuss respectively the applicability of wavelets in the analysis of business cycles (US GDP), the evaluation of economic forecasts, and the study of growth patterns in the US GDP, we use the Maximum Overlap Discrete Wavelet Transform (MODWT) to provide a scale dependent determination of turning points. This allows us to point out that the BBQ algorithm is deeply related to a time-frequency decomposition of the time series at a fixed scale. We show that, by construction, the MODWT combines time and frequency analyzes in an optimal way in the sense of saturating the bound of the time-frequency uncertainty principle. For our empirical application, we follow (Drehmann et al., 2012) and use credit and house prices to characterize the financial cycle in 19 OECD countries. We thus contribute not only to the literature of business and financial cycles, but also to growing literature on the applicability of wavelet analysis to finance and economics (Gençay et al., 2001; Wu and Lee, 2015; Yazgan and Özkan, 2015).

2. The Maximum Overlap Discrete Wavelet Transform (MODWT)

2.1. General presentation

We follow Percival and Walden (2006) and Gençay et al. (2001) in their introduction of the MODWT. Let $\boldsymbol{x}$ be a column vector containing a sequence $x_0, x_1, \ldots, x_{N-1}$ of N observations of a real-valued time series. We assume that the observation x_t was collected at time $t\Delta t$, where Δt is the time interval between adjacent observations (e.g. quarterly). The MODWT of level J is a translation invariance transform of $\boldsymbol{x}$ defined by $\tilde{\boldsymbol{w}} = \widetilde{W}\boldsymbol{x}$, where $\tilde{\boldsymbol{w}}$ is a column vector of length $N(J+1)$, and $\widetilde{W}$ is a $(J+1)N \times N$ real-valued matrix defining the MODWT. The vector $\tilde{\boldsymbol{w}}$ contains the transform coefficients, and may be organized into $J+1$ vectors via

$$\tilde{\boldsymbol{w}} = \left[\tilde{\boldsymbol{w}}_1, \tilde{\boldsymbol{w}}_2, \ldots, \tilde{\boldsymbol{w}}_J, \tilde{\boldsymbol{v}}_J\right]^T \tag{1}$$

where $\tilde{\boldsymbol{w}}_j$ of length N and $\tilde{\boldsymbol{v}}_J$ of length N are called, respectively, vectors of wavelet and scaling coefficients. Qualitatively, if we let $\tau_j = 2^{j-1}$, each $\tilde{\boldsymbol{w}}_j$ is associated with changes on a scale $\tau_j\Delta t$ at a localized set of times; i.e. the wavelet coefficients tell us how much a weighted average changes from a particular time period of effective length $\tau_j\Delta t$ to the next. The coefficients in $\tilde{\boldsymbol{v}}_J$ are in turn associated with variations on scales $\tau_{J+1}\Delta t$ and higher; that is, a scaling coefficient is a weighted average on a scale of length $\tau_{J+1}\Delta t$.

The MODWT is an energy preserving transform in the sense that

$$\|x\|^2 = \sum_{j=1}^{J} \left\|\tilde{w}_j\right\|^2 + \left\|\tilde{v}_J\right\|^2 \tag{2}$$

In a wavelet analysis of variance, each individual wavelet coefficient is associated with a band of frequencies and a specific time scale, whereas Fourier coefficients are associated with a specific frequency only.

2.2. Construction of the MODWT

The MODWT matrix $\widetilde{W}$ is made up of $J+1$ submatrices, each of them of size $N \times N$,

$$\widetilde{W} = \left[\widetilde{W}_1, \widetilde{W}_2, \ldots, \widetilde{W}_J, \widetilde{V}_J\right]^T . \tag{3}$$

and may be described in terms of linear filtering operations. Let $h_1 \equiv h_{1,0}, \ldots, h_{1,L_1}$ be a a wavelet filter of even length $L_1 \leq N$. h_1 may be selected from a Daubechies compactly supported wavelet family, such as the Haar wavelet filter $h_{1,0} = 1/\sqrt{2}, h_{1,1} = -1/\sqrt{2}$. By definition, h_1 must sum up to zero, have unit norm and be orthogonal to its even shifts:

$$\sum_{n=0}^{L_1-1-2l} h_{1,n}h_{1,n+2l} = \begin{cases} 1, & l = 0 \\ 0, & l = 1, 2, \ldots, (L_1-2)/2 \end{cases} \tag{4}$$

h_1 is associated with the scale $\tau_1\Delta t$ and works as an approximate high-pass filter with a pass-band given by the interval of frequencies $[1/(4\Delta t), 1/(2\Delta t)]$. h_1 has a low-pass (scaling) complement $g_1 \equiv g_{1,0}, \ldots, g_{1,L-1}$, defined via the quadrature mirror $g_{1,m} = (-1)^{m+1}h_{1,L-1-m}$. Also, let us define the wavelet filter h_j for higher scales $\tau_j\Delta t$ as the inverse discrete Fourier transform of

$$H_{j,k} = H_{1,2^{j-1}k \mod N} \prod_{l=0}^{j-2} G_{1,2^l k \mod N}, \; k = 0, \ldots, N-1 \tag{5}$$

where $H_{1,k} = \sum_{n=0}^{N-1} h_{1,n}e^{-i2\pi nk/N}, k = 0, , \ldots, N-1$ is the discrete Fourier Transform of the wavelet filter h_1 padded with $N - L_1$ zeros, and $G_{1,\,k}$ is defined similarly in terms of g_1. Elements $h_{j,L_j}, h_{j,L_j+1}, \ldots, h_{j,N-1}$ will be equal to zero when