Finance Research Letters 13 (2015) 225–233

Contents lists available at ScienceDirect

Finance Research Letters

journal homepage: www.elsevier.com/locate/frl

Higher order comoments of multifactor models and asset allocation ☆

Kris Boudt [a,b], Wanbo Lu [c,*], Benedict Peeters [d]

[a] Solvay Business School, Vrije Universiteit Brussel, 1050 Brussel, Belgium
[b] Department of Econometrics and Finance, VU University Amsterdam, Amsterdam, Netherlands
[c] School of Statistics, Southwestern University of Finance and Economics, 610074 Chengdu, China
[d] Finvex Group, 1000 Brussel, Belgium

ARTICLE INFO

Article history:
Received 14 October 2014
Accepted 26 December 2014
Available online 9 January 2015

JEL classification:
G110
G120
G150
C580

Keywords:
Factor models
Higher order comoments
Portfolio selection

ABSTRACT

Accurate estimates of the higher order comoments are needed in asset allocation. We derive explicit formulas for the higher order comoments under the assumption that stock returns are generated by a multifactor model and show that this assumption leads to a substantial reduction in the number of parameters to estimate compared to the traditional approach. An out-of-sample analysis of the performance of portfolio allocation criteria that depend on the higher order comoments illustrates the usefulness of the proposed methodology.

1. Introduction

The distribution of asset returns is often asymmetric and heavy tailed. If there is no estimation error, most investors would be willing to sacrifice expected return and/or accept a higher volatility in exchange for a higher skewness and lower kurtosis leading to a lower downside risk (e.g. Ang

☆ The authors would like to thank Jorn De Boeck, Giang Nguyen, and Joshua Ulrich, and the doctiris program from the Brussels region for support, as well as participants of the 5th R/Finance conference in Chicago (2014). Wanbo Lu's research is sponsored by the National Science Foundation of China (71101118) and the Program for New Century Excellent Talents in University (NCET-13-0961).

* Corresponding author.
E-mail addresses: kris.boudt@vub.ac.be (K. Boudt), luwb@swufe.edu.cn (W. Lu), bp@finvex.com (B. Peeters).

http://dx.doi.org/10.1016/j.frl.2014.12.008
1544-6123/

et al., 2006; Harvey and Siddique, 2000). This trade-off between positive preferences for odd moments (mean, skewness) and negative preferences for even moments (variance, kurtosis) can be conveniently summarized into a single objective function using a Taylor expansion of the expected utility function as objective (as e.g. in Jondeau and Rockinger, 2006 or Martellini and Ziemann, 2010) or a portfolio downside risk objective based on the Cornish-Fisher expansion (e.g. Boudt et al., 2013).

The important caveat is that portfolio moments need to be estimated, and that, without restrictions on the data generating process, the number of parameters to estimate becomes quickly very large when the dimension of the investment universe increases. This curse of dimensionality makes the unrestricted estimators of the first four (co) moments almost infeasible for moderately large dimensions. Suppose e.g. that we have a universe of 20 assets, then the number of unique elements in the covariance, coskewness and cokurtosis is 210, 1540 and 8555, respectively. This is clearly an excessive number of parameters compared to the number of observations that are available in realistic applications. As noted by Michaud (1998), the portfolio optimizer acts like an "error maximizer" and amplifies the estimation errors even further when optimizing the portfolio weights and leads to optimized portfolios that are often not well-diversified (Green and Hollifield, 1992).

In this article, we avoid the curse of dimensionality by assuming that the asset returns are generated by multifactor model. The total number of unique elements in the covariance, coskewness and cokurtosis matrix of 20 assets is 83, 112 and 151 for 1, 2 and 3 factors, respectively. The only related papers are Martellini and Ziemann (2010) recommending to use a single factor model assumption and Ghalanos et al. (2015) who model the higher order moments of asset returns assuming the returns can be rewritten as a linear combination of independent factors. Our approach is more general and decomposes the return vector into a linear combination of a lower dimensional set of possibly dependent factors and a vector of residual terms that can be interpreted as independent idiosyncratic factors.

We illustrate the usefulness of the multifactor approach to higher order comoments in an international portfolio context where the investor allocates with the purpose to maximize his expected utility. The universe consists of four equity benchmarks, nine bond indices and five commodity indices. We find that accounting for the higher order moments using a multifactor approach increases out-of-sample average returns, decreases portfolio standard deviations and leads to an important reduction in the portfolio downside risk.

In what follows, we first describe in Section 2 the general higher order moment estimation framework that we propose. This includes our key contribution, which is the derivation of the explicit formula for the higher order comoments when the asset returns are generated by a multifactor model. In Section 3 we then study the out-of-sample performance of portfolios that use these higher order comoments. The major findings are summarized in the conclusions.

2. The higher order comoments of the multifactor model and their application in asset allocation

2.1. Asset allocation, higher order comoments and the factor model

Besides the forecasted mean return, the key input parameters for the portfolio decision that we study are the covariance, coskewness and cokurtosis matrix of the N-dimensional return vector r with mean μ_r, i.e. the comoments corresponding to (i) the products of two returns, i.e. the covariance of assets i and j:

$$\sigma_{i,j} = E[(r_{(i)} - \mu_{r(i)})(r_{(j)} - \mu_{r(j)})], \quad (1)$$

(ii) the products of three returns, i.e. the coskewness of assets i,j and k:

$$\phi_{i,j,k} = E[(r_{(i)} - \mu_{r(i)})(r_{(j)} - \mu_{r(j)})(r_{(k)} - \mu_{r(k)})], \quad (2)$$

and (iii) the products of four returns, i.e. the cokurtosis of assets i,j,k and l:

$$\psi_{i,j,k,l} = E[(r_{(i)} - \mu_{r(i)})(r_{(j)} - \mu_{r(j)})(r_{(k)} - \mu_{r(k)})(r_{(l)} - \mu_{r(l)})]. \quad (3)$$

It will reveal useful to stack all these comoments into a $N \times N$ covariance matrix Σ, $N \times N^2$ coskewness matrix Φ and $N \times N^3$ cokurtosis matrix Ψ of the return vector r, i.e.: