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The Mills Ratio and the behavior of redeemable bond prices in the Gaussian structural model of corporate default

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ABSTRACT

This paper shows that forward default intensities in the Black and Cox (1976) model of corporate default can be expressed in terms of the Mills Ratio (Mills, 1926). The behaviour of the forward default intensity and hence the survivorship functions then follows from inequalities that are satisfied by this ratio. This allows me to analyze the effect of the firm's distance to default, growth rate and volatility upon the value of its debt. These results can be used to analyze the comparative static properties of other models of corporate default and perhaps other first passage time models.

1. Introduction

The structural model of corporate default analyses the effect of accounting information on asset prices and is well known in the corporate finance literature. The basic model was developed by Leland (1994), Black and Cox (1976) and Leland and Toft (1996) has been extensively used in studying optimal capital structure, default and the pricing of corporate securities. This model has a Gaussian structure which furnishes closed form solutions. Important extensions include Duffie and Lando (2001) who allow for asymmetric information and Hackbarth et al. (2006) who allow the dynamics of the firm to depend upon macroeconomic conditions that can change the regime. Anderson and Sundaresan

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(2000) and others have successfully used the structural approach to explain the prices of perpetual debt and equity in terms of accounting information. Leland (1994) provides closed form solutions for equities and perpetuities, while Black and Cox (1976) and Leland and Toft (1996) provide closed form solutions for redeemable bonds. The solutions for redeemable bonds depend upon partial differential equations or first passage times that are much harder to analyze than the ordinary differential equations used to value perpetuities. For example, while Leland (1994) shows that an increase in a firm's asset value increases the value of its perpetual debt, this has not yet been demonstrated for the value of redeemable debt. The reduced form approach which models the hazard rate directly has typically been used for pricing these instruments.

This paper shows that forward default intensities in the basic Gaussian model can be expressed in terms of the Mills Ratio, which is the ratio of the complementary cumulative distribution function to the density function for a Gaussian random variable (Mills, 1926). The comparative statics of the forward default intensity and hence the survivorship function then follow directly from inequalities which Gordon (1941) shows are satisfied by the Mills Ratio. These inequalities imply that increases in the firm's asset value and its growth rate both reduce the forward default intensity and increase the value of redeemable debt. An increase in volatility unambiguously increases the forward default intensity for a firm with a non-negative growth rate. If the growth rate is negative, the effect of volatility depends upon whether it is larger in absolute value than the distance to default divided by the forward maturity. If so, increases in volatility unambiguously reduce the forward default intensity irrespective of the initial values of volatility and other parameters. If not, the effect is ambiguous: it can switch sign as the values of volatility and other parameters change.

This paper is set out as follows. Section 2 describes the model and derives the formula for the forward default intensity. Section 3 defines the Mills Ratio and shows how it can be used to obtain the comparative static results. The final section discusses these results, suggesting that they might be extended to other Gaussian models of corporate default, as well as other areas in economics and finance in which first passage time problems occur.

2. Default intensities and survivorship

Default intensities play the same role in the defaultable asset markets as forward rates do in the default-free markets. We can abstract from the effect of default-free forward rates by working with the survivorship function $p(m)$ which shows the probability under the risk-neutral measure of the firm surviving from an initial time of 0 until time $m > 0$. The probability of default over this interval is: $\pi(m) = 1 - p(m)$. Assuming that $p(m)$ is differentiable then the forward default intensity for time m is defined as:

$$h(m) = \frac{1}{(1-\pi(m))}\frac{\partial \pi(m)}{\partial m} = \frac{-\partial p(m)}{p(m)\partial m} \tag{1}$$

which may be integrated to get back:

$$p(m) = \exp\left[-\int_0^m h(u)du\right] \tag{2}$$

Vanilla coupon bond prices can also be written in terms of these functions. For example, if the safe rate of return is a constant r, a bond with face value 1, default value θ, coupon c and maturity m has the value:

$$B(c,\theta,m) = e^{-rm}\{1-\pi(m)[1-\theta]\} + c\int_0^m e^{-ru}[1-\pi(u)]du. \tag{3}$$

2.1. Default behavior in the Black and Cox (1976) model

Reduced form debt pricing models model the default intensity directly, as a single stochastic process. However structural debt pricing models model this mathematically in terms of accounting