Optimal multi-period consumption and investment with short-sale constraints

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ABSTRACT

This article examines agents' consumption-investment problem in a multi-period pure exchange economy where agents are constrained with the short-sale of state-dependent risky contingent claims. In equilibrium, agents hold options written on aggregate consumption in their optimal portfolios. Furthermore, under the specific case of quadratic utility, the optimal risk-sharing rule derived for the pricing agent leads to a multifactor conditional consumption-based capital asset pricing model (CCAPM), where excess option returns appear as factors.

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1. Introduction

Classical asset pricing theories assume no restrictions on short-sales. However, as discussed in Jones and Lamont (2002), regulations and procedures administered by various exchanges, underwriters, and individual brokerage firms can mechanically impede short selling. A recent example is the short-sale ban by four Eurozone countries on banking stocks during the height of Eurozone debt crisis in August 2011. By incorporating this economically plausible constraint into individuals' optimal

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consumption-investment problem, we first derive agents' optimal portfolios in a general equilibrium framework, and then test its implications for pricing.

The economic intuition behind borrowing, nonnegative wealth and short-sale constraints goes back to the notion of bounded credit by Dybvig and Huang (1988). Cox and Huang (1989) and Merton (1990) consider a similar problem, and solve the individual agent's optimal investment-consumption problem under nonnegativity constraints within a partial equilibrium framework. Vanden (2004) further solves the problem in a single period general equilibrum framework. We examine optimal sharing rules in general equilibrium using a multi-period securities market framework where agents have the possibility to trade time-event contingent claims. Setting the problem in a multi-period framework is more appealing, because the real-life practice is to trade securities through dynamically managed portfolios.

More specifically, we assume a multi-period pure exchange economy with a single consumption good where heterogeneous agents trade time-event contingent claims at each period. In this economy, agents are constrained with the maximum amount that they can borrow and short sell state-contingent claims. In other words, agents can borrow, buy, sell and short-sell time-event contingent claims at time t, given that they are able to pay-off their debts and do not reach negative wealth levels at time $t + 1$. Restricting short-sales helps agents avoid states of insolvency, and allow them come back to the economy next period with the ability to repay their debts.

The solution to this discrete-time optimization problem implies that agents optimally hold options in their portfolios at each period and state in time. More specifically, in equilibrium, the pricing agent's optimal consumption incorporates the aggregate consumption, and a bundle of short-lived options written on the aggregate consumption with different strike prices. Via their leverage property and nonlinear payoffs, options help agents replicate their optimal consumption patterns, which is otherwise not possible through trading time-event contingent claims under short-sale constraints. Thus, an efficient allocation of resources is achieved at each point in time by holding and dynamically trading short-lived options.

The finding that agents optimally hold options in their equilibrium portfolios in the presence of short-sale constraints is in line with recent stream of studies which argue that the introduction of traded options represents an economically important relaxation of short-sale constraints.[3] According to these studies, options help bypass short sale constraints on financial markets by enabling investors with negative information about the underlying asset to take synthetic short positions in the option market.[4] Similarly, in our setting, options help agents bypass short-sale constraints and avoid reaching negative wealth levels, which is not possible through trading of state-contingent claims. Furthermore, our findings lend support to the theory that options are non-redundant securities, and have an allocational role in the economy when markets are incomplete due to market frictions.[5]

Our paper is also closely related to the literature on heterogeneous preferences. In our model, for the case of quadratic utility, differences in endowments will imply that investors with different budget constraints will have different relative risk aversions. Due to differences in initial endowments, when the level of aggregate consumption goes down, there will be agents who will not consume because of their binding constraints. This has an impact on risk sharing rules of remaining agents because they can not share risks with those who do not consume anymore. As aggregate risk gets concentrated on fewer people, this implies higher risk and different optimal wealth processes for agents whose constraints do not bind. This is why some investors might hold options in their optimal portfolios in order to hedge that increased risk. Indeed, Dumas (1989) shows that the distribution of wealth plays the role

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[3] See theoretical contributions of Chen et al. (2002), Duffie et al. (2002), Hong and Stein (2003), and Scheinkman and Xiong (2003), and empirical studies by Diether et al. (2002), Geczy et al. (2002), Chen and Singal (2003).

[4] Although similar in spirit, this paper differs from the above studies by focusing on short-sale of state-contingent Arrow–Debreu securities rather than short-sale of long-lived complex securities. However, since complex securities can be created using bundles of state-contingent claims, the short-sale constraint of the type studied in this paper can be viewed as an indirect constraint on short-sale of more complex securities such as common stocks or bonds.

[5] See Grossman (1988), Back (1993), Grossman and Zhou (1996), Basak and Croitoru (2000), Liu and Pan (2003), Vanden (2004), and Buraschi and Jiltsov (2006) for articles that motivate options trading under different market imperfections.