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Closed form valuation of American chained knock-in options

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ABSTRACT

In this paper, we study pricing of American chained knock-in option. Chained barrier option is a new type of barrier option with two barrier levels. A knock-in American chained barrier option under a trigger clause is an option contract in which the option holder receives an American knock-in option conditional on the underlying asset price breaching a specified barrier level. We derive analytic valuation formulas for knock-in American chained options under the Black–Scholes pricing framework by using reflection principle and formulas for knock-in American options. Furthermore, we present some numerical solutions and plots of the value of knock-in American chained options.

1. Introduction and preliminaries

Barrier options are one of the most popular path-dependent derivatives in various markets, particularly in OTC markets and FX markets, since barrier options are cheaper and they are more liquid than vanilla options. Therefore, many people have researched on barrier options so far. Reiner and Rubinstein (1991) derived a formula for barrier options. Rich (1994) also got the value of barrier option under mathematical framework.

As barrier options have become popular, a variety of new barrier options which consist of more complicated contract emerged. For example, Geman and Yor (1996) derived the price of double barrier options using Laplace transforms. Heynen and Kat (1994) studied partial barrier options as well. Especially, we note the paper by Jun and Ku (2012) which handle a special type of barrier option with two barrier levels. Different from usual double barrier options, the second barrier level for this option is activated only when the underlying asset hits the first barrier level. Therefore, option is worthless if it does not cross two barrier levels in a specific order. These kind of options were first treated by Pfeffer (2001) and Li (1999), and extended more generally by Jun and Ku. These types of options have recently become popular in the Japanese over-the-counter equity and foreign exchange derivative markets.

Jun and Ku named such options as chained barrier options and studied on pricing them when two barrier levels are given by exponential functions of time (Jun and Ku, 2013). Furthermore, Jun and Ku (2015) approximated the value of American chained barrier options using the approximation method of Ingersoll (1998), which is based on constant exercise policies of barrier options.

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In this paper, we prove mathematically that the value of knock-in American chained barrier options are expressed in terms of the value of knock-in American options by using reflection principle of Brownian motions. That is, we derive the integral equation satisfied by American chained barrier options. This leads to more accurate valuation of American chained barrier options. Our method is also useful for valuing European chained options as well.

This paper is organized as follows. In Section 1, we review analytic valuation formula for knock-in American option derived by Dai and Kwok (2004). Section 2 presents a proof which shows the value of knock-in American chained barrier options can be represented in terms of the value of knock-in American options by using the reflection principle of Brownian motions. In Section 3, we get numerical solutions for the derived integral equations using recursive integration method proposed by Huang et al. (1996), and compare these results with other existing methods.

The usual assumptions for the Black–Scholes option pricing framework are adopted in this work. The stock price S_t is assumed to follow the risk neutral process:

$$dS_t = (r - q)S_t dt + \sigma S_t dW_t \tag{1.1}$$

where r is the risk-free interest rate, σ and q are the volatility and dividend yields of S, respectively, and W_t is a one-dimensional standard Brownian motion on a filtered probability space $(\Omega, (\mathcal{F}_t), \mathbb{P})$, where $(\mathcal{F}_t)_{t\geq 0} \equiv \mathbb{F}$ is the natural filtration generated by W_t.

Consider a knock-in American call option where the knock-in trigger clause entitles the holder to receive an American call option with strike price K when underlying asset S passes the threshold level D. Then, in view of optimal stopping problems, the value $CA_{id}(t, S, K, D)$ of the American down-and-in call option with expiration T is expressed by

$$CA_{id}(t, S, K, D) = \sup_{t\leq\tau\leq T} E^P[e^{-r(\tau-t)}(S_\tau - K)^+ \mathbf{1}_{\{\min_{0\leq\gamma\leq\tau} S_\gamma \leq D\}} \mid S_t = S] \tag{1.2}$$

under the risk-neutral measure P. Similarly, the value of $PA_{iu}(t, S, K, U)$ of the American up-and-in put option with expiration T is given by

$$PA_{iu}(t, S, K, U) = \sup_{t\leq\tau\leq T} E^P[e^{-r(\tau-t)}(K - S_\tau)^+ \mathbf{1}_{\{max_{0\leq\gamma\leq\tau} S_\gamma \geq U\}} \mid S_t = S] \tag{1.3}$$

Dai and Kwok (2004) proved the following theorems.

Theorem 1.1. The value of American knock-in barrier options.

(1) *American down-and-in barrier call options: CA_{id}*
Let $CE_{id}(t, S, K, D)$ and $CA_{id}(t, S, K, D)$ denote the pricing function of the European down-and-in barrier call options and American down-and-in barrier call options, respectively, both with down-and-in barrier D, strike price K and expiration T. For $D \leq K\max(1, r/q)$,

$$CA_{id}(t, S, K, D) = \left(\frac{D}{S}\right)^{\frac{2(r-q)}{\sigma^2}-1}\left[CA\left(t, \frac{D^2}{S}, K\right) - CE\left(t, \frac{D^2}{S}, K\right)\right] + CE_{id}(t, S, K, D) \tag{1.4}$$

where $CE(t, S, K)$ and $CA(t, S, K)$ are the pricing functions of European vanilla call options, American vanilla call options, respectively, both with strike price K and expiration T.
Especially, if $D \leq K$, CA_{id} can be simplified as follows:

$$CA_{id}(t, S, K, D) = \left(\frac{D}{S}\right)^{\frac{2(r-q)}{\sigma^2}-1} CA\left(t, \frac{D^2}{S}, K\right) \tag{1.5}$$

(2) *American up-and-in barrier put options: PA_{iu}*
Let $PE_{iu}(t, S, K, U)$ and $PA_{iu}(t, S, K, U)$ denote the pricing function of the European up-and-in barrier put option and American up-and-in barrier put option, respectively, both with up-and-in barrier U, strike price K and expiration T. For $U \geq K\min(1, r/q)$,

$$PA_{iu}(t, S, K, U) = \left(\frac{U}{S}\right)^{\frac{2(r-q)}{\sigma^2}-1}\left[PA\left(t, \frac{U^2}{S}, K\right) - PE\left(t, \frac{U^2}{S}, K\right)\right] + PE_{iu}(t, S, K, U) \tag{1.6}$$

where $PE(t, S, K)$ and $PA(t, S, K)$ are the pricing functions of European vanilla put options, American vanilla put options, respectively, both with strike price K and expiration T.
Especially, if $U \geq K$, PA_{iu} can be simplified as follows:

$$PA_{iu}(t, S, K, U) = \left(\frac{U}{S}\right)^{\frac{2(r-q)}{\sigma^2}-1} PA\left(t, \frac{U^2}{S}, K\right) \tag{1.7}$$

Dai and Kwok (2004) analyzed the value of knock-in put options PA_{iu} according to the value of U and K(for put options). They showed that for $U < K\min(1, r/q)$, there is no analytic formula for PA_{iu} as (1.6) in Theorem 1.1. Therefore, in this paper, we assume that $U \geq K\min(1, r/q)$ (for put option) and $D \leq K\max(1, r/q)$ (for call option).