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Effect of lifetime uncertainty on consumption/investment with luxury bequest motives

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ABSTRACT

We study optimal consumption/investment of a retiree who has luxury bequest motives and faces the nonnegative bequest constraint. His lifetime is uncertain but actuarially fair life insurance-annuity policies are available. We obtain a closed form solution by using a dynamic programming method, and investigate the effects of lifetime uncertainty and the presence of life insurance-annuity on the consumption/investment policies.

1. Introduction

Bequest motives are natural factors in modeling behavior of economic agents, as mentioned in Carroll (2002). Studies such as Menchik (1980), Auten and Joulfaian (1996), Hurd and Smith (2002) also find that bequests are luxury goods. Ding et al. (2014) introduced a luxury bequest preference into the portfolio choice model by Merton (1969), Merton (1971). They investigated the effects of age on the optimal asset allocation, and found that luxury bequests increase the portfolio share when compared to the homothetic preference, but the difference is small at the outset of retirement. However, in their model, lifetime is certain and neither life insurance nor annuity is considered. In reality, life insurance and annuity account for large parts of households' financial assets. For instance, Guiso et al. (2002) reported that more than 50% of households' assets are held in insurance-annuity in the Netherlands. In such circumstance, considering a lifetime uncertainty and insurance-annuity is indispensable for better understanding of people's behavior.

We investigate optimal consumption-investment and insurance-annuity policies of a retiree having luxury bequest motives in the presence of lifetime uncertainty. As usual, we consider an infinitesimal term life insurance policy which may be negative. In such a case, it is considered as a purchase of infinitesimal annuity as in Yaari (1965), Bernheim (1991), Dybvig and Liu (2010). We find an optimal strategy in a closed form and examine its properties. We claim that the luxury bequests are indeed luxury in the sense of Wachter and Yogo (2010) by showing that the expenditure share for bequests rises in total consumption, or equivalently, in wealth. We compare our results with those in the homothetic model, and find that luxury bequest motives increase not only the investment proportion as in Ding et al. (2014) but the consumption as well.

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We examine the effect of lifetime uncertainty on consumption/investment by a numerical comparison with the fixed lifetime model (Ding et al., 2014). It shows that both consumption and risky investment are higher at all wealth level when lifetime is uncertain. We then look at the age effect on investment after controlling for the current wealth level. In a similar analysis in Ding et al. (2014) of the deterministic lifetime model, they state that "at the outset of retirement it is not important in practice to account for luxury bequests when allocating assets." However, our analysis of uncertain lifetime model suggests that higher proportion of investment in risky assets is necessary to prepare bequests before the termination comes unexpectedly.

2. Optimal consumption/investment

There are two assets in the financial market: One is risk-free with constant rate $r > 0$ and the other is risky. The price S_t of the risky asset evolves according to a geometric Brownian motion with a constant drift $\alpha > r$ and volatility $\sigma > 0$; $dS_t = S_t(\alpha dt + \sigma dw_t)$, $t \ge 0$, where $(w_t)_{t\ge 0}$ is a standard Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$. Let $(\mathcal{F}_t)_{t\ge 0}$ be the augmentation under $\mathbb{P}$ of the natural filtration generated by $(w_t)_{t\ge 0}$.

We have an economic agent whose uncertain lifetime is given by a random variable τ with intensity h which is independent of $(\mathcal{F}_t)_{t\ge 0}$. We consider an actuarially fair infinitesimal term life insurance policy: The agent enters a life insurance contract by paying the premium rate p_t at time t, which is an $\mathcal{F}_t$-predictable process satisfying $\int_0^\tau |p_s| ds < \infty$ a.s. In compensation, if the agent dies at time t then the insurance company pays the insurance benefit p_t/h. As usual, we allow negative insurance by considering it as the agent entering an infinitesimal term annuity contract.

The dollar amount π_t invested in the risky asset and the consumption rate $c_t \ge 0$ at time t are $\mathcal{F}_t$-progressively measurable processes satisfying $\int_0^t \pi_s^2 ds < \infty$ and $\int_0^t c_s ds < \infty$ a.s. for all $t \ge 0$. Then, the wealth process x_t is described by

$$dx_t = (\alpha - r)\pi_t dt + (rx_t - c_t - p_t)dt + \sigma \pi_t dw_t, \quad t < \tau.$$

We assume that the agent exhibits constant relative risk aversion toward consumption and that he has luxury bequest motives described by a bequest function in Ding et al. (2014): The agent has the objective function

$$E\left[\int_0^\tau e^{-\beta t}\frac{c_t^{1-\delta}}{1-\delta}dt + e^{-\beta\tau}\theta^\delta \frac{(\theta a + x_\tau)^{1-\delta}}{1-\delta}\right] \tag{2.1}$$

where β is the subjective discount rate, $\delta \neq 1$ is a positive utility curvature parameter[1], $\theta = \phi/(1-\phi)$ is the transformation of a parameter $\phi \in (0, 1)$ which is the marginal propensity to bequeath in Lockwood (2014), and $a > 0$ is a bequest utility parameter. As in Ding et al. (2014), we impose the nonnegative bequest constraint

$$x_\tau = x_{\tau-} + \frac{p_\tau}{h} \ge 0 \tag{2.2}$$

so that $\theta a + x_\tau \ge \theta a > 0$. We call $(c_t, \pi_t, p_t)_{0\le t\le\tau}$ an admissible policy if it satisfies (2.2). The agent endowed with initial wealth $x_0 > 0$ chooses an admissible consumption/investment and life insurance-annuity policy to maximize the objective function (2.1). The maximized objective function, $V(x_0)$, is called the value function. In order to make the problem well-posed(Merton, 1969; Karatzas et al., 1986; Choi et al., 2003) we assume that

$$K := \frac{\beta + h - (r+h)(1-\delta) - \frac{\gamma(1-\delta)}{\delta}}{\delta} > 0 \quad \text{where} \quad \gamma := \frac{1}{2}\left(\frac{\alpha - r}{\sigma}\right)^2. \tag{2.3}$$

To describe the value function and an optimal policy, we introduce several functions: Let $\lambda_- < -1$ and $\lambda_+ > 0$ be roots of the quadratic equation $\gamma\lambda^2 - (r - \beta - \gamma)\lambda - (r+h) = 0$, and $\rho_\pm := 1 + \lambda_\pm$. The condition (2.3) is equivalent to $1 + \delta\lambda_- < 0$. Define

$$\mathcal{X}(y) = \begin{cases} \dfrac{h\theta a^{1+\delta\lambda_+}}{\gamma\lambda_+(1+\delta\lambda_+)(\lambda_+-\lambda_-)}y^{\lambda_+} + \dfrac{1+h\theta}{K}y^{-\frac{1}{\delta}} - \dfrac{h\theta a}{r+h}, & y < a^{-\delta}, \\ \dfrac{h\theta a^{1+\delta\lambda_-}}{\gamma\lambda_-(1+\delta\lambda_-)(\lambda_+-\lambda_-)}y^{\lambda_-} + \dfrac{1}{K}y^{-\frac{1}{\delta}}, & y \ge a^{-\delta}, \end{cases}$$

$$\mathcal{J}(y) = \begin{cases} \dfrac{h\theta a^{1+\delta\lambda_+}}{\gamma\rho_+(1+\delta\lambda_+)(\lambda_+-\lambda_-)}y^{\rho_+} + \dfrac{1+h\theta}{K}\dfrac{y^{-\frac{1-\delta}{\delta}}}{1-\delta}, & y < a^{-\delta}, \\ \dfrac{h\theta a^{1+\delta\lambda_-}}{\gamma\rho_-(1+\delta\lambda_-)(\lambda_+-\lambda_-)}y^{\rho_-} + \dfrac{1}{K}\dfrac{y^{-\frac{1-\delta}{\delta}}}{1-\delta} + \dfrac{h\theta}{\beta+h}\dfrac{a^{1-\delta}}{1-\delta}, & y \ge a^{-\delta}. \end{cases}$$

Since $\mathcal{X}(y)$ is strictly decreasing, there exists its inverse function $\mathcal{Y}(x)$ from $(0, \infty)$ onto $(0, \infty)$.

We now present the main result.

[1] $\log c$ corresponds to the case of $\delta = 1$. Similar results can be obtained as well.