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Some new results about optimal insurance demand under uncertainty

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ABSTRACT

The aim of this paper is to investigate the optimal insurance demand of a risk-averse agent who is faced with background uncertainty. The preferences of the agent are represented by two-moment, mean-standard deviation utility functions. By the comparative statics, we find that under the assumption of decreasing absolute risk aversion (DARA), the changes of background uncertainty have effects on optimal insurance demand.

1. Introduction

The topic of how insurance demand responds to the change of risk has attracted much interest of many researchers in the fields of economics and finance since long (see e.g., Briys et al. (1993); Broll et al. (1995); Broll and Eckwert (1999)). And the optimal insurance demand problem is especially hot (see e.g., Demers and Demers, 1991; Hadar and Seo (1990) and Dionne and Gollier (1992)). More recently the comparative statics about optimal insurance demand in $(\mu,\sigma)-$space have attracted much more interest of many researchers. According to Battermann et al. (2002), a risk-averse agent reduces his demand for insurance upon an increase in the risk of an insurable wealth loss if and only if the elasticity of his risk aversion with respect to the standard deviation of wealth is greater than unity. Bonilla and Ruiz (2014) have recently argued that the risk-averse individuals will always reduce their demand for insurance when the expected value of the insurable loss decreases. Eichner and Wagener (2014) argue that insurance demand goes down when the expected size of insurable losses decreases or insurance premia increases if the elasticity of risk aversion with respect to expected wealth exceeds -1. The results all of the above only considered the effects of the changes of random loss itself, but didn't consider other factors. In this paper, we define a new model, which is different to the model in Battermn et al. (2002). In fact, decision makers always face many kinds of background uncertainty, such as economic background, policy environment etc. These factors often have

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impacts on decision makers. So, in this paper, we extend the model of Bonilla and Ruiz (2014) by adding a background uncertainty risk. This kind of model is especially useful in the analysis of optimal demand for price index insurance. Our study proceeds as follows: Section 2 presents the new model and discuss the comparative statics effects of some parameter changes. Section 3 presents the main results of the paper. Section 4 concludes with some brief remarks.

2. The model

Like in Bonilla and Ruiz (2014), we define Y_i as a random variable which belongs to the random set Y. As in Sinn (1983) and Meyer (1987), we also suppose that each element of Y differs from one another only by location and scale parameters. Hence, each Y_i has a finite mean and standard deviation denoted by μ_i and σ_i respectively. Let $X = \frac{Y_i - \mu_i}{\sigma_i}$, then every Y_i has the same distribution function as $\mu_i + \sigma_i X$, no matter which was selected to define X. Therefore, the expected utility from Y for any agent with utility function $u(\cdot)$ can be denoted by

$$Eu(Y) = \int_a^b u(\mu + \sigma x) dF(x) =: \phi(\mu, \sigma). \tag{1}$$

The interval $[a, b] \subseteq R$ with $a < b$ is the support of X, and the distribution function of X is F. According to the results of Sinn (1983) and Meyer (1987), it is true that $\phi_u > 0$, $\phi_\sigma < 0$, $\phi_{\mu u} < 0$, where the subscripts denote the partial derivatives.

For the convenience of later proof, we list a series of equivalence between ϕ and u (see Meyer, 1987 or Wagener, 2003):

$$u'(y) > 0 \Leftrightarrow \phi_\mu(\mu, \sigma) > 0; \tag{1.1}$$

$$u''(y) < 0 \Leftrightarrow \phi_\sigma(\mu, \sigma) < 0; \tag{1.2}$$

$$u'''(y) > 0 \Leftrightarrow \phi_{\mu\sigma}(\mu, \sigma) > 0; \tag{1.3}$$

$$u''(y) < 0 \Leftrightarrow \phi \text{ is strictly concave: } \phi_{\mu\mu} < 0, \phi_{\sigma\sigma} < 0, \text{ and } \phi_{\mu\mu}\phi_{\sigma\sigma} - \phi_{\mu\sigma}^2 > 0; \tag{1.4}$$

$$\left(-\frac{u''(y)}{u'(y)}\right)' < 0 \Leftrightarrow \phi_\mu \phi_{\mu\sigma} - \phi_\sigma \phi_{\mu\mu} > 0. \tag{1.5}$$

Now we consider the following model about insurance demand, which is different to the model in Eichner and Wagener (2014), and Bonilla and Ruiz (2014).

Assume that a risk averse agent has an initial wealth w, and faces an insurable risky loss L and a background uncertainty ε. The mean and the standard deviation of the loss L are represented by μ_L and σ_L. The mean and the standard deviation of the background uncertainty is zero and σ_ε respectively. Let α be the coinsurance rate. In this coinsurance problem we allow for arbitrary correlations between insurable loss L and background uncertainty ε. The loss L can be insured at constant marginal costs $(1+\lambda)\mu_L$, where $\lambda > 0$ is a fixed loading factor. Then the final wealth equals to $y(\alpha) = \varpi - (1-\alpha)L - (1+\lambda)\alpha\mu_L + \varepsilon$. Hence, the mean μ_y and variance σ_y^2 of $y(\alpha)$ amount to$\varpi - (1+\alpha\lambda)\mu_L$, $(1-\alpha)^2\sigma_L^2 + \sigma_\varepsilon^2 - 2(1-\alpha)\mathrm{cov}(L, \varepsilon)$ respectively. In (μ, σ)-space, the maximum utility problem about coinsurance rate for the agent can be represented as

$$\max_\alpha \phi(\mu_y(\alpha), \sigma_y(\alpha)), \tag{2}$$

where $\mu_y(\alpha) = \varpi - (1+\alpha\lambda)\mu_L$, $\sigma_y(\alpha) = \sqrt{(1-\alpha)^2\sigma_L^2 + \sigma_\varepsilon^2 - 2(1-\alpha)\mathrm{cov}(L, \varepsilon)}$.

Now, for later reference, we list some formulas about $\sigma(\alpha)$

$$\frac{\partial\sigma(\alpha)}{\partial\alpha} = \frac{(\alpha-1)\sigma_L^2 + \mathrm{cov}(L, \varepsilon)}{\sigma(\alpha)}; \tag{2.1}$$

$$\frac{\partial\sigma(\alpha)}{\partial\sigma_L} = \frac{(1-\alpha)^2\sigma_L}{\sigma(\alpha)} > 0; \tag{2.2}$$

$$\frac{\partial\sigma(\alpha)}{\partial\sigma_\varepsilon} = \frac{\sigma_\varepsilon}{\sigma(\alpha)} > 0; \tag{2.3}$$

$$\frac{\partial^2\sigma(\alpha)}{\partial\alpha^2} = \frac{\sigma_L^2\sigma_\varepsilon^2(1-\rho^2(L, \varepsilon))}{\sigma^3(\alpha)} > 0; \tag{2.4}$$

$$\frac{\partial^2\sigma(\alpha)}{\partial\alpha\partial\sigma_L} = \frac{(\alpha-1)\sigma_L[(\alpha-1)^2\sigma_L^2 + 3(\alpha-1)\mathrm{cov}(L, \varepsilon) + 2\sigma_\varepsilon^2]}{\sigma^3(\alpha)}; \tag{2.5}$$

$$\frac{\partial^2\sigma(\alpha)}{\partial\alpha\partial\sigma_\varepsilon} = -\frac{\sigma_\varepsilon[(\alpha-1)\sigma_L^2 + \mathrm{cov}(L, \varepsilon)]}{\sigma^3(\alpha)}; \tag{2.6}$$