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Shortage function and portfolio selection: On some special cases and extensions

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ABSTRACT

The shortage function has recently been introduced in portfolio selection theory for measuring efficiency. In this paper we focuss on the case of shortselling. We show that, in such a case, the shortage function can be computed in closed form. Some issues concerning duality are also analyzed. We also analyze the case of a riskless asset.

1. Introduction

Distance functions, have been introduced by Shephard (1953) for efficiency measurement either in input or output orientation.

At the same time, Markowitz (1952, 1959) has formulated the mean–variance model, a mathematical approach for determining the optimal risk-return trade-off for portfolio selection. This approach is based upon quadratic programming. However, its computational cost was very high. Hence, Sharpe (1963) had developed the simplified diagonal model and later formulated the capital asset pricing

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model (CAPM) with Lintner (1965). Markowitz (2008) criticized the relation between risk and excess returns described by the linear model due to Sharpe and Lintner. He argued that different expected returns might surely be obtained from the same risk structure.

Nevertheless, the mean–variance approach is the cornerstone of portfolio management and risk assessment. The purpose of this paper is to consider some issue in the measurement of portfolio efficiency.

This contribution extends the analysis proposed in Briec et al. (2004, 2007) where a general framework was introduced that is based upon the *shortage function* a concept introduced by Luenberger (1995) in microeconomic analysis. Transposed in a portfolio optimization context, this function looks for possible simultaneous improvement of return and reduction of risk in the direction of a vector g. In this paper, we make other investigations about measures in the case where there is short-selling. In particular, it is shown that the shortage function can then be computed in closed form. Moreover, it is also established that one can obtain a duality result linking the indirect mean–variance utility and the shortage function. We also consider the case of a riskless asset and provide a computation of the shortage function in closed form in such a case.

This paper is organized as follow. In Section 2, we succinctly present the basic tools of the portfolio management approach proposed in Briec et al. (2004). Section 3 focusses on the shortage function, duality properties and the indirect utility function under relaxed assumptions. In Section 4, we focuss on the Sharpe model (Sharpe, 1963, 1964). A closed form for the return-oriented shortage function is established in presence of a riskless asset. In Section 5, the case of short selling is considered and we show that a closed form of the shortage function can be established using Lagrangian calculus. We also provide duality results in closed form under the presence of short selling. A concluding section outlines conclusions and possible extensions.

2. Efficient frontier and portfolio management

This section introduces main ideas of the portfolio selection problem. Let us consider a market with n financial assets. Note $E[R_i]$ for $i = 1, \ldots, n$ the expected return of the asset i and Ω the covariance matrix of these assets such that $\Omega_{i,j} = Cov[R_i, R_j]$ for $i,\ j \in \{1, \ldots, n\}$. A portfolio is an combination of one or more of these assets. Their proportions may be represented by the vector $x = (x_1, x_2, \ldots, x_n)$ with $\sum_{i=1}^{n} x_i = 1$ and $x_i \geqslant 0$ if short sales is not allowed.

It is assumed throughout the paper that economic constraints (Pogue, 1970; Rudd and Rosenberg, 1979) are linear functions of the asset weights. Thus, the set of the admissible portfolios may be written as follow:

$$\Im = \left\{ x \in \mathbb{R}^n : \sum_{i=1}^{n} x_i = 1, x \geqslant 0 \} \right\}. \tag{2.1}$$

If short-sales are allowed, the admissible set of portfolio is

$$\Im_0 = \left\{ x \in \mathbb{R}^n : \sum_{i=1}^{n} x_i = 1 \} \right\}. \tag{2.2}$$

In such a case, the set of admissible portfolios is extended from the unit simplex to an unbounded hyperplane. In general, the constraint $Ax \geqslant 0$ represents the economic and managerial constraints the manager must deal with.

The return of portfolio x is $R(x) = \sum_{i=1}^{n} x_i R_i$. The expected return and its variance can be defined as follows: $E[R(x)] = \sum_{i=1}^{n} x_i E[R_i]$ and $Var[R(x)] = \sum_{i,j}^{n} x_i x_j Cov[R_i, R_j]$, respectively. For the sake of simplicity, we shall denote in the remainder of the paper $V[R(x)] = Var[R(x)]$. In addition, we consider the map $\Phi : \Im_0 \longrightarrow \mathbb{R}^2$ defined by

$$\Phi(x) = (V[R(x)], E[R(x)]). \tag{2.3}$$

The return and the variance are continuous in x. Hence $\Phi(\Im_0)$ is a compact subset of $\mathbb{R}^2$. Following the Markowitz approach, the subset $\Phi(\Im_0)$ is important to identify the efficient portfolios. However, it is

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