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Asset pricing with skewed-normal return

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ABSTRACT

Despite the fact that it is easy to see intuitively why skewness and coskewness should matter for asset pricing, it is difficult to build a model that links analytically skewness premia to *deep* structural parameters governing preferences and the distribution of shocks. This paper takes up the challenge and studies the effect of *skewness* and *coskewness* on asset valuation. To reach this important goal, asset returns skewness is modeled with promising Azzalini's [1985. Scandinavian Journal of Statistics 12, 171–178] skew-normal distribution. With this assumption, we are now able to derive explicit expressions of assets skewness premiums and to shed a new light on asset valuation.

1. Introduction

Since Samuelson (1970) and Kraus and Litzenberger (1976), skewness is believed a key determinant of asset returns. Intuitively, skewness matters because risk averse investors should have a preference for positively skewed portfolios. Derived risk premium expressions do not however always easily distinguish the contributions of volatility and skewness on expected returns. This is because de-

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rived risk premiums are reduced form expressions of deeper structural parameters governing expected returns. For instance, Harvey and Siddique (2000) derive a conditional three-moment CAPM model from the assumption that the stochastic discount factor is a quadratic function of the market return. This has the advantage of not making premiums depend on the peculiarity of any specific distribution. There are however, drawbacks. Changing the skewness of a stochastic process does not generally leave untouched its variance. In other words, second and third moments of a distribution are tied by a common set of *structural* parameters. This makes it difficult to isolate the influence that skewness has on expected returns in Harvey and Siddique's (2000) type of framework. The absence of a distributional assumption also means that empirical analysis cannot be made with fully efficient (minimum variance) estimation methods. This paper introduces an asset pricing model, based upon Azzalini's (1985) Skew-Normal (*SN*) distribution. It shows how Azzalini's distribution can be used to derive explicit expressions for risk premiums that untangle the separate contribution of volatility and skewness. The paper also highlights the contribution of *Idiosyncratic coskewness* in asset pricing. The *SN* distribution, which has hardly been used in financial economics, is a natural starting point to study the effect of skewness on asset pricing. The normal distribution is a special case of the *SN* distribution, making direct comparisons with the extensive literature possible. Perhaps more importantly, the skewness and coskewness of joint skew-normal variates have explicit expressions. We exploit these features to obtain explicit expressions for the skewness premium and the standard market premium.

The paper is organized as follows. Section 2 reviews the main features of the skew-normal distribution. Section 3 derives the restrictions imposed by the Euler equation of optimal portfolio diversification in a skew-normal environment. This section gives explicit expressions for the skewness premium and the standard market premium. Finally, the conclusion are drawn in Section 4.

2. The skew-normal distribution

We begin with a brief presentation of the *skew-normal* distribution. Let $\phi(\cdot)$ and $\Phi(\cdot)$ be the density and cumulative distribution functions of the *standard normal* distribution. Azzalini (1985) shows that

$$f(z;\alpha) = 2\ \phi(z)\ \Phi(\alpha\, z) \qquad \text{with} \quad (-\infty < z < \infty) \quad \text{and} \quad \alpha \in \mathbb{R} \tag{1}$$

is a density function for a standardized univariate skewed z variate. Azzalini (1985) denominates (1) the *skew-normal* distribution ($SN(\alpha)$) with shape parameter α. Fig. 1 illustrates $SN(\alpha)$ for three different values of α. Negative skewness arises with negative α while positive α leads to positive skewness.

The skew-normal distribution shares many formal properties with the normal distribution. Noteworthy, $SN(\alpha)$ becomes the standard normal when α is zero. Also, squares and/or sum of squares of

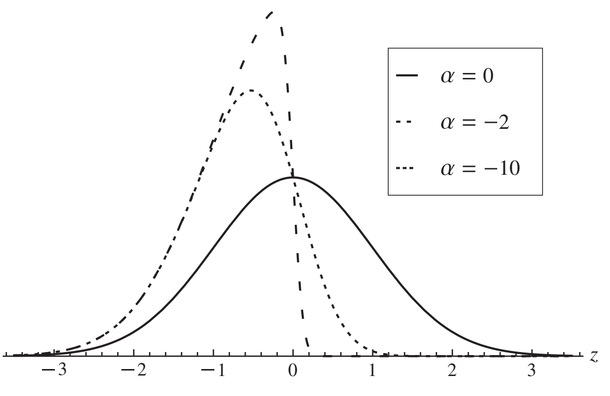

Fig. 1. The skew-normal distribution.