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Composition of robust equity portfolios

Jang Ho Kim [a], Woo Chang Kim [a], Frank J. Fabozzi [b,*]

[a] *Department of Industrial and Systems Engineering, Korea Advanced Institute of Science and Technology (KAIST), Daejeon, Republic of Korea*
[b] *EDHEC Business School, Nice, France*

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ABSTRACT

Robust portfolios resolve the sensitivity issue identified as a concern in implementing mean–variance analysis. Because robust approaches are not widely used in practice due to a limited understanding regarding the portfolios constructed from these methods, we present an analysis of the composition of robust equity portfolios. We find that compared to the Markowitz mean–variance formulation, robust optimization formulations form portfolios that contain a fewer number of stocks, avoid large exposure to individual stocks, have higher portfolio beta, and show low correlation between weight and beta of the stocks composing the portfolio. These properties are also found for global minimum-variance portfolios.

1. Introduction

Markowitz (1952, 1959) presents a portfolio optimization method for finding the optimal portfolio based on the risk-return tradeoff using the expected return, variance, and covariance of asset returns. Despite the model's simplicity, it is still used in asset allocation. One of the main criticisms of the approach is its lack of robustness to the input values (Michaud, 1989; Best and Grauer, 1991a, 1991b; Broadie, 1993; Chopra and Ziemba, 1993). Among many approaches to increase the robustness of mean–variance portfolios, robust optimization focuses on finding the optimal portfolio under the worst-case situation. Although there have been many studies on formulating robust portfolio

* Corresponding author.
E-mail addresses: janghokim@kaist.ac.kr (J.H. Kim), wkim@kaist.ac.kr (W.C. Kim), frank.fabozzi@edhec.edu (F.J. Fabozzi).

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problems (Lobo and Boyd, 2000; Goldfarb and Iyengar, 2003), robust optimization has not been adopted by practitioners as an approach to manage portfolio risk.

One of the reasons that limit the use of robust portfolios is the weak understanding on the characteristics of portfolios formed from robust optimization methods. Recently, some studies analyze whether robust portfolios are more dependent on fundamental factors when compared against mean–variance portfolios. Kim et al. (2012) derive an analytical explanation for the robust formulation with an ellipsoidal uncertainty set that explains why portfolios with high robustness become closer to the portfolio whose variance is maximally explained by fundamental factors. Furthermore, Kim et al. (forthcoming-a) provide empirical evidence under various settings that robust portfolios from robust optimization show higher correlation with the Fama–French three factors (Fama and French, 1993). They both conclude that increased robustness leads to higher dependency on fundamental factors. Although they reach a meaningful conclusion at the portfolio-level, a complete analysis will also require examining the behavior of individual stocks that are included in robust portfolios. For example, Tütüncü and Koenig (2004) experiment with indices of five to 10 asset classes to document the change in the allocation of assets as they move along the efficient frontiers and find that robust portfolios contain fewer asset classes. Pflug and Wozabal (2007), in contrast, find that robustness leads to more diversified portfolios but their analysis includes only six stocks. Even though these two studies provide some preliminary observations, they use only a limited number of candidate assets and focus on the diversification level because the primary objectives of these papers were not to investigate the composition of robust portfolios. Although Lu (2011) also examines the level of diversification along with performance measures such as the rate of growth of wealth and transaction cost, he focuses on assessing two robust approaches that use separable and joint uncertainty sets without including a comparison to the mean–variance model.

Therefore, in this paper, we present a thorough analysis of the composition of robust equity portfolios. By focusing on robust portfolio optimization methods based on worst-case optimization, we find several characteristics of robust equity portfolios and observe these properties in the global minimum-variance portfolio.

The organization of the paper is as follows. Section 2 briefly reviews robust portfolio formulations. Section 3 introduces the data and test model used for analyzing those robust models and the results are included in Section 4. Section 5 summarizes our analysis on robust equity portfolios and Section 6 concludes.

2. Robust portfolio formulations

Among several robust approaches, we focus on robust formulations based on worst-case optimization. Worst-case robust problems search for the optimal portfolio while assuming the worst possible situation within a predetermined set for the uncertain inputs (Lobo and Boyd, 2000; Goldfarb and Iyengar, 2003). The robust behavior of investors can be explained by the Ellsberg paradox (1961) where decision makers are shown to be highly affected by their aversion to uncertainty. The maxmin expected utility decision rule describes this behavior because the minimum utility of each case is compared (Gilboa and Schmeidler, 1989).

In order to construct portfolios with the same level of risk aversion under mean–variance and robust models, we use the following objective function throughout the analysis,

$$\min_{\omega \in \Omega} \omega' \sum \omega - \lambda \mu' \omega$$

where ω is the portfolio weights, $\sum$ is the covariance matrix of asset returns, μ is the expected asset returns, λ is the risk-seeking coefficient, and Ω is the set of feasible portfolio weights. For the robust formulations, the objective function is first maximized within an uncertainty set to find the worst case. We only consider uncertainty in expected return of assets because it is known to have a stronger effect on portfolio return than the variance or covariance of asset returns (Chopra and Ziemba, 1993). In our study, the uncertainty set for expected return of assets is considered to follow a box or an ellipsoidal shape.[1]

[1] For the covariance matrix of estimation errors in the ellipsoidal uncertainty set, we assume the off-diagonal terms to be zero (Stubbs and Vance, 2005).