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Simulated testing of nonparametric measure changes for hedging European options

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ABSTRACT

We test the accuracy and hedging performance of the deltas given by a range of nonparametric measure changes. The nonparametric models accurately estimate deltas across a number of asset price dynamics. The optimal nonparametric measure change displays superior estimation bias, which depends on how the models capture the stylised features of the dynamics, moneyness, and time-to-expiry. Differences in estimation error appear negligible. The optimal measure change produces superior static hedging outcomes compared to the Black–Scholes model. Differences in dynamic hedging outcomes are negligible.

1. Introduction

Stutzer (1996) introduces the Canonical nonparametric model for pricing European options, and Haley and Walker (2010) derive the Euclidean (EU) and Empirical Likelihood (EL) models as alternative measure changes.

Alcock and Gray (2005) derive the Canonical model deltas and show their accuracy under geometric Brownian motion. We show across a range of asset price dynamics that various nonparametric models derived from members of the Cressie and Read (1984) family of divergence functions accurately estimate European option deltas. The optimal nonparametric measure change displays improved estimation bias over the Stutzer's (1996) Canonical method, with this bias depending on how the models capture the stylised features of the dynamics, moneyness and time-to-expiry. Differences in estimation error between the models appears negligible.

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Alcock and Gray (2005) report superior dynamic hedging outcomes by the Canonical model compared to the Black and Scholes (1973) model under Heston (1993) dynamics. However, when tested against market data, Gray et al. (2007) report the Canonical model outperforming only for puts, and Alcock and Smith (2012) report similar outcomes between the nonparametric and Black–Scholes models, with a naive model that uses constant instead of risk-neutral probabilities often outperforming. We show across a range of dynamics that the naive model produces superior static but inferior dynamic hedging outcomes, the optimal nonparametric measure change produces superior static hedging outcomes compared to the Black–Scholes model, and differences in hedging outcomes between the nonparametric and Black–Scholes models vanish for dynamic hedging.

2. The nonparametric pricing formulae

Consider pricing a European option with T years to expiry, strike K and underlying whose value is S. The nonparametric models value European options in the risk-neutral framework by first estimating a sample $R=(R_1,\ldots,R_N)$ of asset returns at expiry and assigning it the measure $\pi^{\mathbb{P}}=(1/N,\ldots,1/N)$. Second, a risk-neutral measure $\pi^{\mathbb{Q}}=(\pi_1^{\mathbb{Q}},\ldots,\pi_N^{\mathbb{Q}})$ is estimated by minimising a divergence function[1] subject to the constraint $\mathbb{E}^{\mathbb{Q}}[R]=e^{rT}$. Third, for a given $\pi^{\mathbb{Q}}$, the prices of European options are $C=e^{-rT}\sum_{n=1}^{N}\pi_n^{\mathbb{Q}}(SR_n-K)^+$ and $P=e^{-rT}\sum_{n=1}^{N}\pi_n^{\mathbb{Q}}(K-SR_n)^+$, where r is the risk-free rate.

The Canonical model of Stutzer (1996) uses the Kullback–Leibler (KL) divergence $KL(\pi^{\mathbb{Q}})=\sum_{n=1}^{N}\pi_n^{\mathbb{Q}}\log(N\pi_n^{\mathbb{Q}})$, whose minimisation yields

$$\pi_n^{KL}=\frac{\exp(\lambda^{KL}R_ne^{-rT})}{\sum_{n=1}^{N}\exp(\lambda^{KL}R_ne^{-rT})},\quad \text{where } \lambda^{KL}=\min_{\lambda\in\mathbb{R}}\left\{\sum_{n=1}^{N}\exp(\lambda(R_ne^{-rT}-1))\right\}.$$

Haley and Walker (2010) generalise the Canonical (KL) model, noting that the KL divergence is just one member of the Cressie and Read (1984) family:

$$CR_\alpha(\pi^{\mathbb{Q}})=\frac{2}{\alpha(1+\alpha)}\sum_{n=1}^{N}\frac{1}{N}\left[\left(\frac{1}{N\pi_n^{\mathbb{Q}}}\right)^{\alpha}-1\right],\quad \alpha\in\mathbb{R}.$$

The limit $\alpha\to-1$ gives the KL divergence. Haley and Walker (2010) explore the EU ($\alpha=-2$) and EL ($\alpha\to0$) divergences, given by $EU(\pi^{\mathbb{Q}})=\frac{1}{2N}\sum_{n=1}^{N}(N\pi_n^{\mathbb{Q}}-1)^2$ and $EL(\pi^{\mathbb{Q}})=-\frac{1}{N}\sum_{n=1}^{N}\log(N\pi_n^{\mathbb{Q}})$, and show that

$$\pi_n^{EL}=\frac{1}{N}\frac{1}{1+\lambda^{EL}(R_ne^{-rT}-1)},\quad \text{where } \frac{1}{N}\sum_{n=1}^{N}\frac{R_ne^{-rT}-1}{1+\lambda^{EL}(R_ne^{-rT}-1)}=0,$$

$$\text{and}\quad \pi_n^{EU}=\frac{1}{N}(1-\lambda^{EU}e^{-rT}(R_n-\mathbb{E}[R])),\quad \text{where } \lambda^{EU}=\frac{\mathbb{E}[R]e^{-rT}-1}{\frac{N-1}{N}e^{-2rT}\mathbb{V}\mathrm{ar}[R]}.$$

The EL model prices options the most accurately when R correctly models the asset dynamics (Haley and Walker, 2010; Alcock and Smith, 2012).

We also investigate the divergences given by $\alpha=1$ (Pearson's χ^2) and $\alpha=2$: $CHI(\pi^{\mathbb{Q}})=\frac{1}{2N}\sum_{n=1}^{N}\left[(N\pi_n^{\mathbb{Q}})^{-1}-1\right]$ and $CR_2(\pi^{\mathbb{Q}})=\frac{1}{6N}\sum_{n=1}^{N}\left[(N\pi_n^{\mathbb{Q}})^{-2}-1\right]$. We employ an interior-point scheme to minimise these divergences.

3. The nonparametric delta formulae

The derivation of the KL deltas by Alcock and Gray (2005) are independent of $\pi^{\mathbb{Q}}$, so $\Delta_C=\frac{\partial C}{\partial S}=e^{-rT}\sum_{n=1}^{N}\pi_n^{\mathbb{Q}}R_n1_{\{SR_n>K\}}$ and $\Delta_P=\frac{\partial P}{\partial S}=-e^{-rT}\sum_{n=1}^{N}\pi_n^{\mathbb{Q}}R_n1_{\{K>SR_n\}}$ for each $\pi^{\mathbb{Q}}$ (1_A is the indicator function of A).

[1] Divergence functions quantify the statistical or probabilistic distance between $\pi^{\mathbb{P}}$ and $\pi^{\mathbb{Q}}$.