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What is the correct meaning of implied volatility?

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Abstract

This paper presents a closed-form solution for the valuation of European options under the assumption that the excess returns of an underlying asset follow a diffusion process. In light of our model, the implied volatility computed from the Black–Scholes formula should be viewed as the volatility of excess returns rather than as the volatility of gross returns. Using the SPX and the OMX options data, we test whether implied volatility obtained from Black-Scholes option price explains the volatilities of excess returns better than gross returns, even though the result is not statistically significant.

JEL classification: G12; G13; G14; C13

Keywords: Option; Implied volatility; Realized volatility; Excess return; Gross return

1. Introduction

A forward measure has been one of the useful tools for bond pricing since it was introduced by Jamshidian (1989). Before Geman et al. (1995) derived an option pricing formula by changing the risk neutral measure into a forward measure, Kim (1992) introduced an option pricing model using an underlying asset price normalized by the bond price with the same maturity of an option. The two ideas are the same, but the probability measure needs not to be changed in Kim's model.

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An interesting feature of Kim's model is that a specific stochastic process for interest rates needs not to be identified, in contrast with Merton (1973), Rabinovitch (1989), and Amin and Jarrow (1992) even though Kim was able to obtain an option pricing formula under a stochastic process for interest rate matched with an option's expiration. Also, the number of parameters is the same as the Black and Scholes's (1973) (called BS hereafter) model so that implied volatilities of options can be calculated. We claim that this implied volatility represents the realized volatility of excess returns and not that of gross returns.

To examine these empirical implications, we test whether implied volatility calculated from our model explains the realized volatility of excess returns better than that of gross returns. For empirical investigation, the SPX and the OMX options data are used. The OMX options data which cover the period of highly volatile interest rates allow us to examine the effect of volatile interest rates on the relation between implied volatility and volatility of excess returns.

The analysis reveals that implied volatility has a slightly stronger association with the volatility of excess log returns than of gross returns. This result is robust over the SPX and the OMX sample data.

The rest of the paper is organized as follows. Section 2 describes the model, Section 3 presents empirical results, and Section 4 concludes.

2. The model

Under a continuous time economy with a complete market, we evaluate a European call option C with a strike price K expiring at T. An underlying asset price and a zero-coupon bond price paying 1 unit at maturity T are denoted by $S(t)$ and $B(t,T)$, respectively. It is assumed that the price of an underlying asset (we call stock hereafter) denominated by a zero-coupon bond price $S(t)/B(t,T)$, denoted by $F(t,T)$, follows a log–normal process,[1] i.e.,

$$\frac{\mathrm{d}F(t,T)}{F(t,T)} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W(t), \tag{1}$$

where $\mathrm{d}W(t)$ is a Wiener process, μ is the instantaneous expected rate of returns of a stock price normalized by a bond price and σ is the standard deviation. Formula (1) implies that the forward price follows a log–normal process. Moreover, the volatility in (1) means the standard deviation of excess log returns of a spot price since the volatility of $\mathrm{d}F(t,T)/F(t,T)$ is the same as that of $\mathrm{d}\log F(t,T)$ by Ito's lemma and

$$\mathrm{d}\log F(t,T) = \log\frac{F(t,T)}{F(t-\mathrm{d}t,T)} = \log\frac{S(t)}{S(t-\mathrm{d}t)} - \log\frac{B(t,T)}{B(t-\mathrm{d}t,T)}.$$

It is known that the option price is homogeneous of degree 1 in two assets, a stock and a bond. Then the option price is

$$C\big(t, S(t), B(t,T)\big) = C\left(t, \frac{S(t)}{B(t,T)}, 1\right)B(t,T). \tag{2}$$

Let us define $V(\tau, F)$ by the normalized option price in (2) as follows:

$$V(\tau, F) \equiv \frac{C(\tau, S(t), B(\tau))}{B(\tau)},$$

[1] In Geman et al. (1995), the forward price is defined by $\mathrm{d}F(t)/F(t) = \sigma\,\mathrm{d}W_F(t)$ without drift because it is a martingale under forward risk neutral measure.