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Some properties of subjective probabilities induced by optimal expectations

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ABSTRACT

In this paper, we examine the properties of subjective probabilities induced by optimal expectations. We show that investors who follow optimal expectations underweigh small probabilities and overweigh large probabilities in a simple binary economy. This indicates that the subjective probabilities induced by optimal expectations are incompatible with experimentally observed results under the rank dependent probability assumption.

1. Introduction

Experimental observations have revealed that people cannot assess the objective probabilities of future outcomes and that we use subjective probabilities for the evaluation of future events. Rank dependent probability models developed by Quiggin (1982) and Tversky and Kahneman (1992) are among the most widely used models for representing subjective probabilities. They describe subjective probabilities using probability weightings that distort objective probabilities. There are many experimental studies, such as those of Gonzalez and Wu (1999), Prelec (1998),[1] and others, that explore subjective probabilities under rank dependent probability models. These studies indicate that

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[1] Prelec (1998) provides the axioms for inverse S-probability weightings.

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we have subjective probabilities determined by inverse S-shaped probability weightings. In other words, we overweigh small probabilities and underweigh large probabilities.

Because rank dependent probability models assume that we have subjective probabilities that are exogenously determined, they cannot explain why we use distorted subjective probabilities in our decisions under risk. This naturally leads us to ask why we overweigh small probabilities and underweigh large probabilities. In a recent study, Brunnermeier and Parker (2005) proposed the concept of optimal expectations, in which subjective probabilities are chosen to maximize lifetime well-being. It is interesting to examine whether subjective probabilities induced optimal expectations are compatible with those determined by inverse S-shaped probability weightings in rank dependent probability models. In this paper, we examine some properties of subjective probabilities induced by optimal expectations in a simple binary state economy. It turns out that, contrary to inverse S-shaped probability weightings, decision makers who follow optimal expectations underweigh small probabilities and overweigh large probabilities.[2]

The paper is organized as follows. Section 2 formally describes our model. In Section 3, we present some properties that subjective probabilities possess under optimal expectations. All proofs relevant to these properties are in Appendix A. The final section contains some concluding remarks.

2. The model

Our model is a simplified version of the optimal expectation advocated by Brunnermeier and Parker (2005) and Gollier (2005). We consider a static Lucas economy, i.e., a pure exchange economy, where an investor with optimal expectations decides her or his optimal portfolio at the initial time $t = 0$ and consumes its realized value at the terminal time $t = 1$. The investor has an unbounded (von Neumann–Morgenstern) utility function u, which is strictly increasing and strictly concave in the terminal consumption. We also assume that the utility function is sufficiently smooth. This assumption guarantees that a subjective probability is continuous with respect to an objective probability, and enables our analysis to avoid unnecessary technical difficulties. Two assets are traded in the economy, a risk-free asset and a risky asset. The risk-free asset is the numeraire in the economy and its net return is normalized to zero without loss of any generality. The risky asset has a return $\mathbf{x} = (x_u, x_d)$, with $x_d < 0 < x_u$. Uncertainty is described by binary states $\{u, d\}$ with the net return $\mathbf{x} = (x_u, x_d)$ and with an objective probability $\mathbf{q} = (q, 1-q)$.

The optimal portfolio of an investor with optimal expectations is given as a solution of a two-stage optimal problem. The first stage is the usual optimal portfolio problem, that is, the investor solves her or his optimal portfolio problem under a given subjective probability. In the second stage, the investor decides on her or his subjective probability that maximizes lifetime well-being, consisting of a convex combination of an objective expected utility and a subjective expected utility.

In the first stage, the investor chooses an optimal allocation between the risk-free asset and the risky asset under a given subjective probability $\mathbf{p} = (p, 1-p)$. Let $\alpha(p)$ denote the optimal portfolio under the subjective probability $\mathbf{p} = (p, 1-p)$. The investor decides the share of the risky asset, $\alpha(p)$ so as to maximize the subjective expected utility defined by:

$$S(p) = \mathbb{E}_{\mathbf{p}}[u(w_0 + \alpha(p)\tilde{x})] = pu(w_0 + \alpha(p)x_u) + (1-p)u(w_0 + \alpha(p)x_d).$$

Let $O(p)$ be the objective expected utility defined by:

$$O(p) = \mathbb{E}_{\mathbf{q}}[u(w_0 + \alpha(p)\tilde{x})] = qu(w_0 + \alpha(p)x_u) + (1-q)u(w_0 + \alpha(p)x_d).$$

Then, the well-being of the investor with optimal expectations is given by a convex combination of the objective expected utility and the subjective expected utility, as follows:

$$W(p) = \kappa S(p) + (1-\kappa)O(p),$$

[2] Because rank dependent probability models, unlike optimal expectations, have nothing to do with belief formation, this result does not necessarily mean that subjective probabilities induced by optimal expectations have the opposite prediction to inverse S-shaped probability weightings under rank dependent probability models.