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Hedging errors with Leland's option model in the presence of transaction costs ☆

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Abstract

Nonzero transaction costs invalidate the Black–Scholes [1973. Journal of Political Economy 81, 637–654] arbitrage argument based on continuous trading. Leland [1985. Journal of Finance 40, 1283–1301] developed a hedging strategy which modifies the Black–Scholes hedging strategy with a volatility adjusted by the length of the rebalance interval and the rate of the proportional transaction cost. Kabanov and Safarian [1997. Finance and Stochastics 1, 239–250] calculated the limiting hedging error of the Leland strategy and pointed out that it is nonzero for the approximate pricing of an European call option, in contradiction to Leland's claim. As a further contribution, we first identify the mathematical flaw in the argument of Leland's claim and then quantify the expected percentage of hedging losses in terms of the hedging frequency and the level of the option strike price.

JEL classification: B23; C15; C61; G13

Keywords: Option pricing and hedging; Transaction costs; Expected percentage of hedging losses

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1. Introduction

Leland (1985) developed a hedging strategy that uses the Black and Scholes (1973) formula with a modified volatility dependent on the rate of transaction costs and the length of trading intervals. He claimed that the modified strategy, inclusive of transaction costs, can be used to approximately replicate the option's payoff as the length of rebalance intervals becomes short. The idea was to offset the transaction costs by properly adjusting the volatility with respect to the length of trading intervals and develop a strategy that converges to the Black–Scholes price as transaction costs become arbitrarily small. Unfortunately, the main theorem (Leland, 1985, p. 1290) is flawed. Intuitively, if the volatility is made arbitrarily large by making the length of rebalance intervals shorter, the hedging strategy converges to a trivial case which holds one share of the underlying stock at any point in time no matter how low the transaction cost rate is. As in Davis and Clark (1994) and Soner et al. (1995), this strategy confirms that the minimum cost for hedging a call option is exactly the price of the stock in the presence of transaction cost and in the framework of continuous trading. This strategy does not provide an exact hedge, since the payoff of holding the underlying stock is greater than that of the call option at maturity as long as the strike price of the option is positive.

Given that option premiums are determined by an optimal hedging strategy, the writer of a call option is interested in knowing when and how a hedging trade is triggered in the presence of transaction costs. Equity put option prices are consistently lower (higher) than the Black–Scholes model prices for in-(out-of-)the-money options, as implied volatilities of out-of-(in-)the-money options are higher (lower) than those of at-the-money ones. The reverse is true for call options. This market phenomenon has been documented as implied volatility skewness, e.g., Rubinstein (1994) and Tompkins et al. (2003). Why would we see such a skewness? It is conceivable that trading frictions, including transactions costs, can partially be a reason. If the underlying stock prices are far above the strike price, the transactions costs should be large, therefore, the premiums of the options should be greater than the Black–Scholes model price.

Option replication has been studied by numerous researchers. In addition to Leland (1985), Boyle and Vorst (1992) designed a perfect hedging strategy in the Cox et al. (1979) binomial model with transaction costs. A perfect hedge is possible due to the assumption of a binomial process for the underlying stock price. Davis et al. (1993) and Edirisinghe et al. (1993) developed a general replicating strategy in the framework of optimization by minimizing the initial cost subject the hedging portfolio payoff to being at least as large as the option's payoff. Constantinides and Zariphopoulou (1999) studied bounds on option prices using a general utility preference. Toft (1996) studied the mean variance tradeoff in option replication. However, his derived result is based on the Leland's adjusted volatility and hence is questionable. Kabanov and Safarian (1997) calculated the limiting hedging error of the Leland strategy and pointed out that it is nonzero for the approximate pricing of an European call option, in contradiction to Leland's claim. As a further contribution to this issue, we first identify the mathematical flaw in the heuristic argument of Leland (1985) and then quantify the expected percentage of hedging losses of a European call option in terms of the hedging frequency and the level of the option strike price.

2. The procedure of dynamic hedging

Before we discuss Leland's work, we review dynamic hedging. Consider a market in which a security is traded with a proportional transaction cost rate k. Assume that an agent sells a derivative security for C_0 with a payoff C_T depending only on the value of the underlying security at