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Likelihood and objective Bayesian modeling of acidity and major ions in rainfall using a bivariate pseudo-Gamma distribution

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ABSTRACT

Modeling the acidity in rainfall at certain locations is a complex task because of different environmental conditions for different rainfall regimes and the large variability in the covariates involved. In this paper, concentration of acidity and major ions in the rainfall in UK is analyzed by assuming a bivariate pseudo-Gamma distribution. The model parameters are estimated by using the maximum likelihood method and the goodness of fit is checked. Furthermore, the non-informative Jeffreys prior for the distribution parameters is derived and a hybrid Gibbs sampling strategy is proposed to sample the corresponding posterior for conducting an objective Bayesian analysis. Finally, related quantities such as the deposition flux density are derived where the general pattern of the observed data appears to follow the fitted densities closely.

1. Introduction

Acid rain, an obstinate environmental problem all over the world today, is affecting the human health, economy, ecosystem and environment adversely. The presence of H^+, SO_4^{2-}, NO_3^- and NH_4^+ in the air is the reason for the acid rain which is produced by fuel combustion in industries and vehicles. According to the US Environmental Protection Agency, acid rain is responsible for the acidification of lakes and streams, ruining the forest soil and trees at high elevations, decaying of building materials and paints, gradual breakdown of statues and sculptures belonging to cultural heritage and degradation of human health as well. These harms could become more adverse if the concentration of the acidity in rain increases.

This paper aims to provide a simple and flexible model for acidity and major ions in rain. Many authors have worked on modeling the acid rain. Zhang et al. (2012) analyze the acid rain patterns in northeastern China using a decision tree method to predict the occurrence of acid rain using geographic position, terrain characteristics, routinely monitored meteorological factors and column concentrations of atmospheric SO_2 and NO_2. Patrinos et al. (1989), Lin et al. (2005) and Anatolaki and Tsitouridou (2009) observe that atmospheric acidic pollution, meteorology, topographic structure and geographic position are significant factors influencing the occurrence of acid rains. Nam et al. (2001) and Singh et al. (2007) believe that meteorological factors affect acid rain as well.

Patrinos et al. (1989) conclude that acid deposition is formed from SO_2 and NO_2 emitted to the atmosphere, largely due to fossil fuels combustion, especially coming from transportation sources. The air pollutants are transformed in the atmosphere into H_2SO_4 and HNO_3 which form condensation nuclei for aerosols and clouds and acidify precipitation; a direct uptake on aerosols and clouds leads to dry and wet acid depositions as well. Hoi et al. (2006) analyze the chemical composition of precipitation to observe the causes of wet deposition in Macau and find that monthly rainfall volume follows the Gamma distribution. Wang and Wang (1996) come to the conclusion that acidic rainfall occurs more frequently due to the effects of atmospheric aerosol loading, alkaline matter content in soil and meteorological conditions in China. Pradofiedler (1990), Seto et al. (1995) and Anatolaki and Tsitouridou (2009) report that the amount of precipitation is a key parameter for acidity due to its scavenging processes that affect the rainfall composition and determine the relationship between acidity and its amount of precipitation. Generally, there is an indirect relationship between the rainfall and the concentration of acidity.

Moreover, Ito et al. (2002) and Ollinger et al. (1993) describe methods for estimating acid rain or the ion concentrations in wet depositions from point data, including multiple regression analysis using geographic position and elevation as independent variables or the ion concentrations in wet depositions. Alternatively, Peck and Schaake (1990) present digital elevation models

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and other topographic models which can be used to represent topographic effects on climate variables. Also Daly et al. (1994) apply the combination of physical and numerical approaches to characterize the climate variables.

Many distributions have been successfully used to parameterize the acidity in rainfall. Among these distributions, the Gamma distribution is a good choice to model acidity in rainfall due to variability in rainfall regimes. It provides a family of flexible distributions, captures some important features of the data such as right-skewness and, after all, leads to an adequate fit in many studies. Different forms of bivariate Gamma distributions are widely used to model different hydrological events, e.g. flood, precipitation, storm, drought, etc. Clarke (1980) applies a bivariate Gamma distribution to an extension of stream flow records that are correlated with longer records of precipitation. Yue (2001) uses a bivariate Gamma distribution in multi flood frequency analysis. Nadarajah (2007) uses a bivariate Gamma model for drought data and derives the distribution of interarrival time and proportion. Yue et al. (2001) review various bivariate Gamma distribution models which are constructed from the Gamma marginals applied frequently in multivariate hydrological event modeling. Cheng et al. (2010) demonstrate a frequency factor based approach for stochastic simulation of a bivariate Gamma distribution which is capable of generating random sample pairs describing marginal densities of random variables as well as their correlation coefficients. Some other types of bivariate Gamma distributions are used in the field of hydrology as well, see Prekopa and Szantai (1978), Nadarajah and Gupta (2006a, 2006b), Nadarajah and Kotz (2006), Loaiciga and Leipnik (2005) and Aksoy (2000).

However, there has been little work in modeling acid precipitation and major ions in rainfall using marginal Gamma distributions with predefined dependence structure. In this study we use a bivariate pseudo-Gamma distribution proposed by Mohsin et al. (2010) to model the concentration of acidity and major ions in rainfall using rainfall data in UK during 1986. This simple and flexible bivariate distribution is the compound distribution of two Gamma distributed random variables, $X \sim Ga(\alpha,\beta)$ and $Y|X \sim Ga(\gamma,\delta X)$. Here, $Ga(\alpha,\beta)$ denotes a Gamma distribution with shape parameter α and scale parameter β. The density of the bivariate pseudo-Gamma distribution is given as

$$f_{(X,Y)}(x,y) = \frac{\beta^{\alpha}\delta^{\gamma}}{\Gamma(\alpha)\Gamma(\gamma)} x^{\alpha+\gamma-1} y^{\gamma-1} \exp\{-x(\beta+\delta y)\}, \quad x,y \geq 0 \qquad (1)$$

with parameters $\alpha,\beta,\gamma,\delta > 0$. Based on this model, the marginal pdfs of the random variables X and Y are

$$f_X(x) = \frac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} \exp\{-\beta x\},$$

$$f_Y(y) = \frac{\beta^{\alpha}\delta^{\gamma}}{B(\alpha,\gamma)} \frac{y^{\gamma-1}}{(\beta+\delta y)^{\alpha+\gamma}}.$$

Here, $\Gamma(\cdot)$ represents the gamma function and $B(\cdot,\cdot)$ represents the beta function. An important characteristic of Eq. (1) is that X and Y are negatively correlated.

The rest of the paper is organized as follows. In Section 2, we describe the data and the study area. In Section 3, we estimate the model parameters by using the ML method and check the goodness of fit. In Section 4, we derive the non-informative Jeffreys prior for the distribution of the parameters. In Section 5, we use Markov chain Monte Carlo simulation techniques to estimate the posterior distribution using Jeffreys's prior. In Section 6, we investigate certain quantities in relation to rain amount, acidity and major ions, e.g. the deposition flux density.

2. Study area and data description

Warren Spring Laboratory UK devised to measure the composition of rainfall all over the country in 1985 using primary and secondary national networks which collected the data on daily and weekly bases, respectively. To obtain reasonably representative data, 47 sites were chosen away from the local sources of pollution. These sites can be seen in Fig. 1 given by Webster et al. (1991). The data was collected from January 1, 1986 to December 31, 1986. The details about this data and the dataset used by Webster et al. (1991) can also be downloaded from the website: http://www.statsci.org/data/general/rainuk.html.

The dataset contains five variables, annual Rain ($mm/a = l/m^2/a$) and the precipitation-weighted annual mean concentrations of four ions: $H^+(\mu eq/l)$, $SO_4^{2-}(\mu eq/l)$, $NO_3^-(\mu eq/l)$ and $NH_4^+(\mu eq/l)$. The measurement of NH_4^+ for site number 35 was not available, so we eliminated this site when analyzing the data for NH_4^+. Since Rain is negatively dependent on H^+, SO_4^{2-}, NO_3^- and NH_4^+, our model Eq. (1) is appropriate in this respect. Table 1 displays the correlations of Rain with H^+, SO_4^{2-}, NO_3^- and NH_4^+.

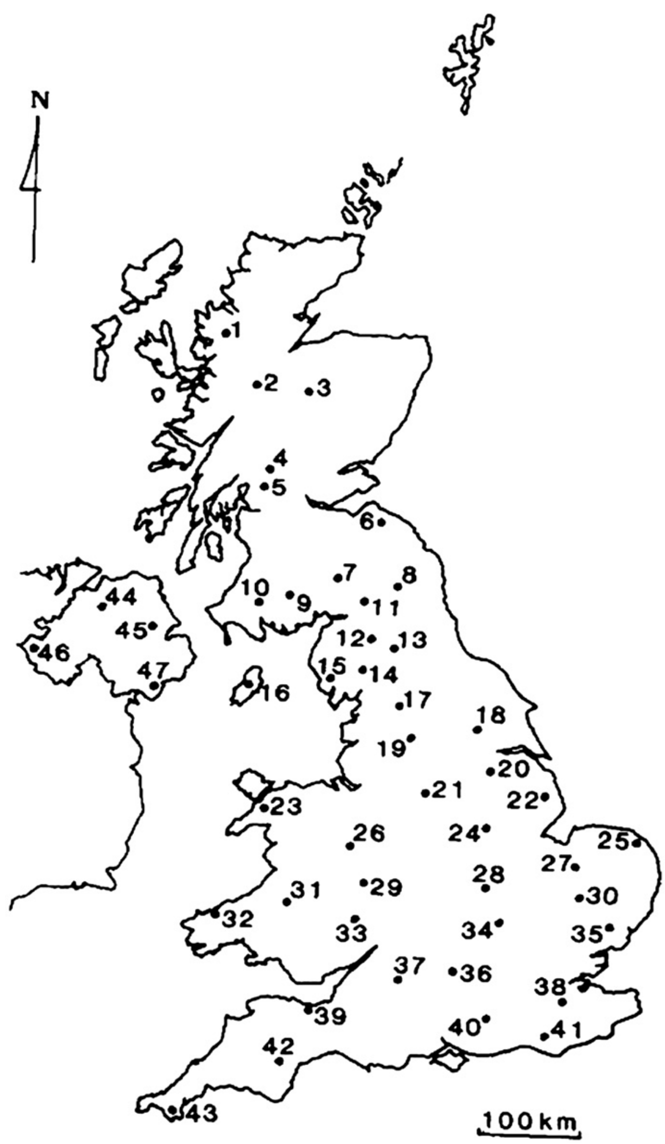

Fig. 1. Map highlighting the selected locations for acid rainfall data.