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Case study

Growth by Optimization of Work (GROW): A new modeling tool that predicts fault growth through work minimization

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ABSTRACT

Growth by Optimization of Work (GROW) is a new modeling tool that automates fracture initiation, propagation, interaction, and linkage. GROW predicts fracture growth by finding the propagation path and fracture geometry that optimizes the global external work of the system. This implementation of work optimization is able to simulate more complex paths of fracture growth than energy release rate methods. In addition, whereas a Coulomb stress analysis determines two conjugate planes of potential failure, GROW identifies a single failure surface for each increment of growth. GROW also eliminates ambiguity in determining whether shear or tensile failure will occur at a fracture tip by assessing both modes of failure by the same propagation criterion. Here we describe the underlying algorithm of the program and present GROW models of two propagating faults separated by a releasing step. The discretization error of these models demonstrates that GROW can predict fault propagation paths within the numerical uncertainty produced by discretization. Model element size moderately influences the propagation paths, however, the final fault geometry remains similar between models with significantly different element sizes. The propagation power of the fault system, calculated from the change in work due to fault propagation, indicates when model faults interact through both soft- and hard-linkage.

1. Introduction

Understanding how faults evolve and interact at different stages of growth is fundamental to mitigating hazard in seismogenic regions. In addition, modeling fracture propagation, including joints and faults, provides insight into subsurface processes controlling the migration of water, ore-hosting fluids, and hydrocarbons. The new modeling tool Growth by Optimization of Work (GROW) uses a global work criterion to predict fracture propagation paths and interaction.

GROW provides an alternative to previous approaches of predicting fracture growth, which include the Hoek–Brown strength criteria (Hoek and Brown, 1980; 1997), the Drucker–Prager criterion (Drucker and Prager, 1952), and the Mogi criterion (Mogi, 1971). Another approach considers the energy release rate, or energy per unit area required to create new fracture surface, *G*. *G* is determined from the stress concentrations at a fracture tip (e.g., Irwin, 1958), which are controlled by the loading on, length and shape of the fracture (e.g., Griffith, 1924; McClintock and Walsh, 1962; Lajtai, 1971). The direction of growth that maximizes *G* predicts the collinear propagation path of opening-mode fractures subject to mode-I loading, such as joints, veins and dikes (e.g., Pollard and Aydin, 1988; and references therein) and the curved paths of opening-mode fractures under mixed-mode loading (e.g., Olson and Pollard, 1991; Cooke and Pollard, 1997; De Bremaecker and Ferris, 2004). *G* also provides a criterion for in-plane growth in shear (modes II or III) (e.g., Irwin, 1958). Although this criterion is applicable to certain materials, it struggles to predict the growth of faults within rock, where failure likely involves multiple, small-scale processes of tensile failure and linkage (e.g., Schultz, 1999; Crider and Peacock, 2004; Savalli and Engelder, 2005), which often result in complex propagation paths.

Previous analyses have predicted the propagation path of faults from the orientation of planes that maximize Coulomb stress (e.g., Crider and Pollard, 1998; Maerten et al., 2002; Olson and Cooke, 2005 ; Pollard and Fletcher, 2005). This approach determines two potential failure planes on which the ratio of shear to normal stress equals the internal coefficient of friction (Jaeger et al., 2007). If the material has anisotropic strength, one of the failure planes could be preferred; however, a robust numerical algorithm that determines which of the two planes fails is not generalizable for isotropic materials (Cooke and Madden, 2014). Furthermore, using a tensile failure criterion and maximum Coulomb stress in parallel can indicate that both tensile and shear failure are possible near a fracture tip, but this approach cannot unambiguously indicate

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Abbreviations: GROW, GRowth by Optimization of Work; W_{ext}, external work; $W_{ext}/\Delta A$, external work divided by new fracture area

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which failure mode is preferred (Cooke and Madden, 2014). Consequently, mode-specific failure criteria struggle to simulate how multiple faults may link and form one continuous structure. This linkage is a primary mechanism by which fault networks evolve (e.g., Crider and Peacock, 2004; and references therein).

GROW uses work minimization as an alternative method of predicting failure orientation and fracture propagation in intact rock. Numerous geologic problems related to the development of crustal faults have been investigated with work minimization, including slip-partitioning in transpressional environments (Jones and Wesnousky, 1992), fault geometry in southern California (Cooke and Kameda, 2002; Olson and Cooke, 2005), and the onset of kink-folding in heterogeneous material (Maillot and Leroy, 2006). Work minimization also has been used extensively to investigate the dynamics of accretionary systems, including the length of new accretionary forethrusts (Gutscher et al., 1998), the temporal evolution of thrusts within accretionary wedges (e.g., Hardy et al., 1998; Del Castello and Cooke, 2007; Cubas et al., 2008), and the distribution of stress in accretionary systems (e.g., Souloumiac et al., 2009, 2010; Yagupsky et al., 2014).

Cooke and Madden (2014) develop a general implementation of work minimization to predict fault and joint propagation paths with various failure modes and along complex propagation paths by assuming that the crust deforms to optimize the external work, W_{ext}, acting on the system. W_{ext} is the integral of the sum of the products of shear traction and displacement, σ_s and u_s, and normal traction and displacement, σ_n and u_n, along the boundaries of the model, B:

$$W_{ext}=\oint_B(\sigma_s u_s+\sigma_n u_n)dB \tag{1}$$

W_{ext} reflects the overall mechanical efficiency of a system, such that the most efficient fault propagation path will produce the maximum change in external work, ΔW_{ext}, which is calculated as the difference in W_{ext} before and after fault propagation. Unlike alternative methods of modeling fault growth, work minimization provides a global approach that considers the energy expended in deformational processes throughout the system. Searching for the most efficient system with work minimization is an optimization problem, and so work *minimization* and work *optimization* may be used interchangeably.

In the following sections, we describe the GROW algorithm and the functionality of Fric2D (Cooke and Pollard, 1997), which GROW repeatedly executes to calculate W_{ext} and thereby model fracture growth. Madden et al. (submitted for publication) verify the GROW algorithm by comparing GROW propagation paths to other predictions of fault growth, and validates this tool by comparing GROW results to laboratory observations. In this paper, we show an application of GROW to two crustal-scale strike-slip faults separated by a releasing step, because a significant advantage of GROW is its application to the mixed-mode propagation of faults, which occurs as the faults interact. We analyze the numerical error of these models produced by discretization. We show that the evolution of W_{ext} closely parallels the propagation paths of the modeled faults, and that changes in W_{ext} can indicate when the modeled faults transfer stress through soft- and hard-linkage.

2. Algorithm

The validated numerical modeling tool GROW, which is available under a free and open source license, models the evolution of a fracture network by iteratively searching for the geometry of fracture growth that maximizes the change in external work due to that growth, ΔW_{ext}, divided by fracture area propagated in each increment of growth, ΔA. We use $\Delta W_{ext}/\Delta A$ because systems with more fracture area are more efficient than systems with less fracture area due to the fact that they can accommodate more strain under the same loading. GROW calculates fracture area by considering fractures to have one unit width because it is a two-dimensional, plane strain modeling tool.

To model fracture propagation, GROW first calculates the initial external work of the system, W_{ext}. Next, GROW identifies the most efficient fracture geometry by (1) deforming the system and calculating $\Delta W_{ext}/\Delta A$ for the first fracture geometry, (2) modifying the geometry in the input file to calculate $\Delta W_{ext}/\Delta A$ for each additional radial potential growth orientation, and (3) identifying the geometry that maximizes $\Delta W_{ext}/\Delta A$. After GROW finds the most efficient geometry, this geometry is set as the new fracture geometry to which GROW now adds potential growth elements. GROW will continue to simulate fracture propagation by repeating the steps above until all of the fractures in the system intersect other fractures or the boundary of the model, or if none of the tips of the fractures fail in tension or shear.

In each propagation step, the most efficient fracture geometry maximizes the magnitude of $\Delta W_{ext}/\Delta A$. However, the boundary conditions of the system will determine whether W_{ext} increases or decreases with fracture growth. When displacements are prescribed to the model boundaries, fracture growth decreases the tractions along the boundaries and W_{ext} decreases (Eq. (1)). If tractions are prescribed to the boundaries, fracture growth increases the displacements of the boundaries and W_{ext} increases. Mixed boundary conditions that include tractions on some model boundaries and displacements on others only provide reliable analysis of W_{ext} if one of these conditions is set to zero. Under these conditions, the model boundaries with either tractions or displacements set to zero do not contribute to W_{ext} (Eq. (1)). For reliable GROW models, all of the non-zero boundary conditions should be either displacement or traction conditions. Either of these loading conditions may be prescribed to analyze the increasing mechanical efficiency of a fracture network with GROW, but displacement boundary conditions typically result in faster execution times and more numerically stable results.

2.1. Fric2D and GROW input

GROW uses the two-dimensional boundary element method numerical modeling tool Fric2D (Cooke and Pollard, 1997) to calculate the stresses and displacements within the deforming fracture system. Fric2D solves the quasi-static equations of deformation on each element to determine the displacement and tractions produced by a given set of boundary conditions and influenced by the fracture geometry. Fractures and boundaries are discretized into linear elements of constant displacement discontinuity. Each linear element defines the edge of a fracture plane that is one unit length (e.g., one meter) in width within the plane strain system. The fractures may open or slip, but may not interpenetrate, in response to tractions or displacements applied to the model boundaries, or from opening or slip along nearby elements.

Following a tension positive sign convention, opening occurs when the normal stress along an element meets or exceeds its tensile strength. Slip occurs when the shear stress meets or exceeds its frictional shear strength, which is the difference between its cohesion and the product of normal stress and the coefficient of friction along the fault. Fric2D uses the penalty method for frictional slip with prescribed shear and normal stiffness along fault elements to ensure that the elements do not interpenetrate (e.g., Cooke and Pollard, 1997; Maerten et al., 2010). Additionally, Fric2D 3.2.7 can simulate slip-weakening behavior along pre-existing fractures and/or potential growth elements (Savage and Cooke, 2010). When an element slips beyond a prescribed slip-weakening distance, the coefficient of friction along that element evolves linearly from its static to its sliding value.