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An impossibility under bounded response of social choice functions [☆]

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ABSTRACT

We introduce a new axiom called *bounded response*, which states that for each "smallest" change of a preference profile, the change in the social choice must be "smallest," if any, for the agent who induces the change in the preference profile. We show that *bounded response* is weaker than *strategy-proofness*, and that *bounded response* and *efficiency* imply dictatorship. This impossibility has a far-reaching negative implication: on the universal domain of preferences, it is difficult to identify a non-manipulability condition that leads to a possibility result.

1. Introduction

We consider a society that chooses one alternative from among a finite set of alternatives, based on agents' preferences. A social choice function (SCF) maps each profile of agents' preferences to an alternative. We propose an axiom called *bounded response*. An SCF satisfies *bounded response* if for each "smallest" change of a preference profile, the change of the social choice must be the "smallest," if any, for the agent who induces the change in the preference profile.

We explain *bounded response* in detail. Given a preference profile $\boldsymbol{R} = (R_1, \ldots, R_n)$, let x be the alternative chosen at $\boldsymbol{R}$. Suppose that one agent, say agent i, exchanges the positions of two consecutively ranked alternatives in R_i. We regard this as the "smallest change" of a preference profile. Let y be the social choice after the agent i's preferences changing. Then, *bounded response* requires either that $x = y$ or that x and y are consecutively ranked in R_i. This implies that a small error in announcing preferences does not make a big difference in the social choice. In addition, it is possible that an agent wavers in deciding which preference to report among similar choices. In such a case, *bounded response* ensures that the agent's decision on which preference to report is not crucial, in the sense that the decision does not make a big difference to the ranks of the social choices. Thus, *bounded response* is a property on stability of social choice and is desirable from the viewpoint of agents. Although it would not be as desirable as widely accepted axioms such as *efficiency*, our result with *bounded response* has important implications, which we discuss shortly.

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Our main result is as follows: an SCF satisfies *bounded response* and *efficiency* if and only if it is dictatorial. This impossibility has interesting and important implications.

First, our main result shows that the property that is distinct from those previously considered, such as *strategy-proofness*, leads to dictatorship.[1] By the Gibbard–Satterthwaite theorem, it is well known that *strategy-proofness* and *efficiency* lead to dictatorship. It can be seen that *bounded response* is weaker than *strategy-proofness*. Thus, *bounded response* is sufficient for the impossibility. Note that *bounded response* is not a condition on incentives to misreport preferences. It simply limits the extent to which the social choice can respond to changes in preferences. Thus, at an agent i's preferences changing from R_i to R_i', it is possible under *bounded response* that the social choice at R_i' is preferable (according to R_i) to the social choice at R_i.

Second, a stronger version of the main result (Proposition 3.3) readily leads to a new interesting impossibility theorem. Following recent research on weaker conditions than *strategy-proofness* (for example, Reffgen, 2011; Carroll, 2012; Sato, 2013; Cho, 2016; Mishra, 2016), we consider a new incentive condition, which we call *weak AM-proofness*. Assume that the options of misrepresentation are restricted to preferences that are adjacent to the true preference, as in Sato (2013). Given a preference profile $\boldsymbol{R}$, let x be the chosen alternative at $\boldsymbol{R}$, and R_i' be a false preference of agent i, which is adjacent to R_i. Let y be the alternative chosen at $(R_i', \boldsymbol{R}_{-i})$, and let z and w be the alternatives whose ranks are exchanged in the passage from R_i to R_i'. *Weak AM-proofness* requires that there exists no R_i' such that the change from x to y $(x \neq y)$ is a noticeable improvement for agent i in the sense that, compared to z or w, the choice x before the change is (weakly) worse and the choice y after the change is (weakly) better (i.e., $y\ R_i\ z\ R_i\ x$ or $y\ R_i\ w\ R_i\ x$). As a straightforward corollary of our arguments for the main result, we can see that *weak AM-proofness* and *efficiency* lead to dictatorship. Even when we allow for profitable misrepresentation, when the degree of the profit is restricted in a different way from Reffgen (2011), we cannot deviate from the impossibility.

A few axioms in the existing literature are related to the idea of how "close" social choices are to each other. The *preference proximity* introduced by Baigent (1987) requires that if a preference profile $\boldsymbol{R}$ is "closer" to some status quo $\boldsymbol{R}^0$ than $\boldsymbol{R}'$, there should be the same relation in the values of the SCF f, that is, $f(\boldsymbol{R})$ is "closer" to $f(\boldsymbol{R}^0)$ than $f(\boldsymbol{R}')$. Our *bounded response* differs from *preference proximity* in that the metric in the set of alternatives is given exogenously in the definition of *preference proximity*, while it is endogenous and depends upon the preference of the deviating agent in *bounded response*. Topological social choice theory considers the continuity of social welfare functions, rather than social choice functions, where the topology in the set of preferences can be defined in various ways.[2] Muto and Sato (2016a) consider an axiom of social welfare functions stating that each "smallest" change of a preference profile leads to the "smallest" change, if any, of the social preference, and prove an impossibility result. This result is about social welfare functions, not SCFs, and shown using a different technique from that used here. In addition, the results are independent of each other in the sense that the results cannot be derived from one another. In a context of "claims problems," Kasajima and Thomson (2016) consider axioms such that the degree of a change of an outcome is bounded by the degree of a change of inputs to a rule.

The remainder of the paper is organized as follows. In Section 2, we introduce notations and definitions, including our main axiom *bounded response*. In Section 3, we present our results. In Section 3.1 we show our main theorem after introducing a technical condition called *same-sidedness*. In Section 3.2, we present an application to *weak AM-proofness*. In Section 3.3, we discuss results when *efficiency* is weakened to *unanimity*. In Section 3.4, we discuss whether our impossibility result holds on restricted domains of preferences. In Section 4, we provide a complete proof of the main theorem. Section 5 concludes the paper.

2. Model

We consider a society consisting of n *agents* in $N = \{1, \ldots, n\}$ where $n \geq 2$. Let X be a finite set of feasible *alternatives* with $|X| = m \geq 3$, and $\mathcal{L}$ be the set of all *linear orders* on X.[3] By definition, $x\ R\ x$ for each $R \in \mathcal{L}$ and each $x \in X$. Each agent $i \in N$ has a *preference* $R_i \in \mathcal{L}$. For each pair of distinct alternatives $x, y \in X$, $x\ R_i\ y$ means that i (strictly) prefers x to y. If each agent i has a preference $R_i \in \mathcal{L}$, the n-tuple $(R_1, \ldots, R_n)$ is denoted by $\boldsymbol{R}$, and if some agent i changes preference from R_i to R_i', the new preference profile is written as $(R_i', \boldsymbol{R}_{-i})$. For each preference $R \in \mathcal{L}$ and each integer k $(1 \leq k \leq m)$, let $r^k(R) \in X$ be the kth-ranked alternative according to R. For each preference $R \in \mathcal{L}$ and each alternative $x \in X$, let $\rho_R(x)$ be the rank of x with respect to R, (i.e., $\rho_R(x) = \big|\{y \in X \mid y\ R\ x\}\big|$). Two alternatives x and y are *adjacent in* $R \in \mathcal{L}$ if they are consecutively ranked in R, (i.e., $|\rho_R(x) - \rho_R(y)| = 1$). Two preferences R and R' are *adjacent* if the only difference between them is the ranks of two adjacent alternatives. If R and R' are adjacent and two distinct alternatives $x, y \in X$ satisfy $x\ R\ y$ and $y\ R'\ x$, the set $\{x, y\}$ is denoted by $A(R, R')$.

A *social choice function* (*SCF*) f is a function from the set of preference profiles $\mathcal{L}^n$ to the set of alternatives X. An SCF is *dictatorship* if there exists $i \in N$ such that $f(\boldsymbol{R}) = r^1(R_i)$ for each $\boldsymbol{R} \in \mathcal{L}^n$. This agent i is called a *dictator*. We introduce a few properties of an SCF. An SCF f satisfies

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[1] See Barberà (2011) for a comprehensive survey on *strategy-proofness*.

[2] See Baigent (2011) for a survey of topological social choice.

[3] A binary relation is a *linear order* if it is complete, transitive, and antisymmetric.