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The Blocking Lemma and strategy-proofness in many-to-many matchings

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ABSTRACT

This paper considers the incentive compatibility in many-to-many two-sided matching problems. We first show that the Blocking Lemma holds for many-to-many matchings under the extended max–min preference criterion and quota-saturability condition. This result extends the Blocking Lemma for one-to-one matching and for many-to-one matching to many-to-many matching problem. It is then shown that the deferred acceptance mechanism is strategy-proof for agents on the proposing side under the extended max–min preference criterion and quota-saturability condition. Neither the Blocking Lemma nor the incentive compatibility can be guaranteed if the preference condition is weaker than the extended max–min criterion.

1. Introduction

Many-to-many two-sided matching models study assignment problems where agents can be divided into two disjoint sets: the set of firms and the set of workers. Each firm wishes to hire a set of workers, and each worker wishes to work for a set of firms. Firms have preferences over the possible sets of workers, and workers have preferences over the possible sets of firms. The assumption that workers may work in more than one firm is not unusual. A physician may have a medical position at a hospital and a teaching position at some university. A faculty member in college may have a part-time position in different places. A many-to-many two-sided assignment problem is to match each agent (a firm or a worker) with a subset of agents from the other side of the market. If a firm hires a worker, we say that the two agents form a partnership. A set of partnerships is called a matching.

The many-to-many matching problem is a natural extension of the one-to-one marriage problem and the many-to-one college admissions problem of Gale and Shapley (1962) with general quotas. The notions with respect to the college admissions problem are commonly generalized to the many-to-many matching model. For a matching problem, the stability of matching is of primary importance. A matching is pairwise-stable if all partnerships occur between acceptable partners (in-

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dividual rationality) and there is no unmatched worker–firm pair that mutually prefer each other to their assigned partners (pairwise blocking). For the marriage problem and the college admissions problem, Gale and Shapley's deferred acceptance algorithms yield stable matchings. Moreover, the stable matching produced by the deferred acceptance procedure is also optimal for agents on the proposing side. That is, every agent on the proposing side is at least as well off under the assignment given by the deferred acceptance procedure as he would be under any other stable assignment. Roth (1984) adapts the deferred acceptance algorithm to the many-to-many matching market and obtains the corresponding optimal stable assignment.

The incentive compatibility of matching is also an important problem. For one-to-one matching problem, Roth (1982) investigates the marriage problem and obtains that the men (resp. women)-optimal matching is strategy-proof for men (resp. women).[2] For a unified model, Hatfield and Milgrom (2005) study the incentive property for matching with contracts. They obtain that the doctor-optimal matching is strategy-proof for doctors under very weak preference assumption (hospitals' preferences satisfy substitutability and the law of aggregate demand). Under the same framework, Hatfield and Kojima (2009) show that the doctor-optimal matching is in fact group strategy-proof for doctors. For many-to-one matching problem, Roth (1985) studies the college admissions problem and shows that, when colleges have responsive preferences, the colleges-proposing deferred acceptance algorithm may not be strategy-proof for colleges, while the students-proposing deferred acceptance algorithm is strategy-proof for students. It is interesting to study the incentive compatibility for agents with multi-unit demand.

For incentive compatibility in many-to-many matching problems, Baïou and Balinski (2000) claim that their reduction algorithm is stable and strategy-proof for agents on one side of the matching market under the max–min criterion condition. Unfortunately, their claim is incomplete. Hatfield et al. (2014) show that the max–min preference criterion is not sufficient for the existence of a stable and strategy-proof matching mechanism even in many-to-one matching markets. As such, it is still an unanswered question on the strategy-proofness in many-to-many matching problems.

This paper considers the (group) strategy-proofness in many-to-many two-sided matching problems under the extended max–min preference criterion and quota-saturability condition. The extended max–min criterion indicates that agents always want to match with as many acceptable partners as possible within their quotas, which means that agents should use their capacity of resources as they can, and focus on the worst partners when ranking different sets of partners. The firms-quota-saturability says that, there is a sufficiently large number of available and acceptable workers in the market such that every firm can hire as many workers as its quota. We show that the firms-proposing deferred acceptance algorithm is (group) strategy-proof for firms if all agents (firms and workers) have the extended max–min preferences and the firms-quota-saturability is satisfied.

In order to obtain the result of strategy-proofness, we first extend the Blocking Lemma to many-to-many matching markets. For one-to-one and many-to-one matching problems, the Blocking Lemma is an important instrumental result, which identifies a particular blocking pair for any unstable and individually rational matching that is preferred by some agents of one side of the market to their optimal stable matching. Its interest lies in the fact that it has been used to derive some key conclusions on matching. Using the Blocking Lemma for one-to-one matching,[3] Gale and Sotomayor (1985) give a short proof for the group strategy-proofness of the deferred acceptance algorithm. For many-to-one matching, the Blocking Lemma holds under responsive preference profile.[4] The responsiveness seems too restrictive to be satisfied. For a weak preference restriction, Martínez et al. (2010) show that the corresponding Blocking Lemma for workers (who have unit demand) holds under substitutable and quota-separable preference.[5] They also note that the Blocking Lemma for firms (which have multi-unit demand) does not hold even under responsive preference. Under the extended max–min preference criterion and quota-saturability condition, Jiao and Tian (2015a) obtain the Blocking Lemma for agents with multi-unit demand in many-to-one matching markets, and then show the strategy-proofness of the deferred acceptance algorithm for agents on the proposing side. It is then interesting to investigate the Blocking Lemma in many-to-many matching markets.

In this paper we show that the extended max–min preference restriction, together with the quota-saturability condition, establishes the Blocking Lemma for many-to-many matchings. As an immediate consequence of the Blocking Lemma, we obtain the strategy-proofness of the deferred acceptance algorithm in many-to-many matchings. In addition, we note by

[2] Dubins and Freedman (1981) show that, under the men (resp. women)-proposing deferred acceptance algorithm, there exists no coalition of men (resp. women) that can simultaneously improve the assignment of all its members if those outside the coalition state their true preferences. This result implies the property of strategy-proofness.

[3] Gale and Sotomayor attribute the formulation of the lemma to J.S. Hwang.

[4] Roth (1985) introduces responsiveness of preference relations for college admissions problems. Specifically, responsiveness means that, for any two subsets of workers that differ in only one worker, a firm prefers the subset containing the most-preferred worker. Formally, we say a firm f's preference relation is responsive if for any w_1, w_2 and any S such that $w_1, w_2 \notin S$ and $|S| < q_f$, we have $S \cup \{w_1\} P(f) S \cup \{w_2\}$ if and only if $\{w_1\} P(f) \{w_2\}$, where w_1, w_2 are the partners of f and S is a set of partners of f. It is easy to obtain that the responsiveness is stronger than the substitutability.

[5] Barberà et al. (1991) propose another concept of separable preference (different from that used by Sotomayor, 1999), which has been extensively used in matching models. See, for instance, Alkan (2001), Dutta and Massó (1997), Ehlers and Klaus (2003), Martínez et al. (2000, 2001, 2004b), Papai (2000), and Sönmez (1996). Based on this condition, Martínez et al. (2010) propose a new concept called quota-separability. Formally, a firm f's preference relation $P(f)$ over sets of workers is quota q_f-separable if: (i) for all $S \subsetneq W$ such that $|S| < q_f$ and $w \notin S$, it implies $(S \cup \{w\}) P(f) S$ if and only if $\{w\} P(f) \varnothing$; (ii) $\varnothing P(f) S$ for all S such that $|S| > q_f$. One can check that the extended max–min criterion introduced in this paper implies (i). The definition of a matching requires that $|\mu(f)| \leq q_f$ for all $f \in F$, and consequently condition (ii) is satisfied. That is, in our setting, the extended max–min criterion is stronger than quota-separability.