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Reaching consensus through approval bargaining [☆]

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A R T I C L E I N F O

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A B S T R A C T

In the Approval Bargaining game, two players bargain over a finite set of alternatives. To this end, each one simultaneously submits a utility function u jointly with a real number α; by doing so she approves the lotteries whose expected utility according to u is at least α. The lottery to be implemented is randomly selected among the most approved ones. We first prove that there is an equilibrium where players truthfully reveal their utility function. We also show that, in any equilibrium, the equilibrium outcome is approved by both players. Finally, every equilibrium is sincere and Pareto efficient as long as both players are partially honest.

1. Introduction

An elementary version of the bargaining problem involves two players with complete information who have to decide on the terms of a possible cooperation. The outcome is either an agreement about such terms, or else a conflict, in the case that no agreement is reached. While dynamic bargaining has been extensively explored and often leads to desirable outcomes (in models à la Rubinstein, 1982), the literature on simultaneous bargaining is scant. It has been argued (see, e.g., Osborne and Rubinstein, 1990) that, not to leave room for renegotiation, the bargaining outcome should be Pareto optimal. Furthermore, if both players are to participate in the bargaining mechanism, then the outcome should not be worse than disagreement.

We design a two-player simultaneous model of bargaining such that, in equilibrium, parties always reach an agreement. Moreover, as long as agents are *partially honest* (Dutta and Sen, 2012) every equilibrium outcome is Pareto efficient. Partial honesty has been recently analyzed by the mechanism design literature and it captures a mild form of preference for honesty. A partially honest agent prefers being sincere over lying whenever sincerity does not lead to a worse outcome.[1] In our model, each player simultaneously approves of a *set of lotteries over the pure alternatives*. A player does so announcing a

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[1] We discuss how our paper relates to the mechanism design literature at the end of the Introduction. We do not attempt to give a review on the bargaining literature and simply refer the reader to Serrano (2008).

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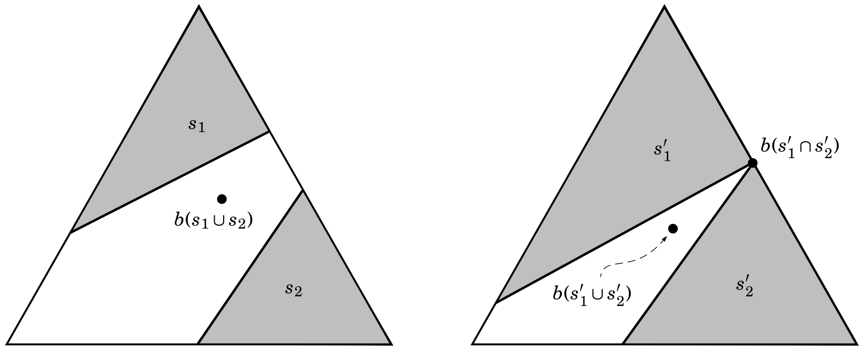

(A) Non-consensual strategy profile. (B) Consensual strategy profile.

Fig. 1. Strategy profiles in the approval bargaining game.

utility function u and a real number α; by doing so she approves of the lotteries whose expected utility according to u is at least α. A player's announcement is sincere if its utility component coincides with her true utility function.[2] And a partially honest player prefers a sincere announcement if she cannot do better by lying.

The outcome induced by strategy profiles is determined as follows. If some lotteries are approved by both players, then the two approved sets intersect. We define the *winning set* to be such an intersection and we say that the winning set is *consensual*. If no lottery is approved by both players, then we define the winning set to be the set of lotteries that are approved by at least one agent. In this case, we say that the winning set is *non-consensual*. Finally, the mechanism selects a lottery at random using the uniform probability over the winning set. The alternative to be implemented is decided by this selected lottery.[3] Thus, in the same vein as Babichenko and Schulman (2015) and Núñez and Laslier (2015), one can think of our model as a reinterpretation of approval voting (Brams and Fishburn, 1983; Laslier and Sanver, 2010) as a bargaining mechanism when there are just two voters.[4] Hence, in the sequel, we refer to our bargaining mechanism as *approval bargaining*.

In some sense, our approval bargaining game is similar to Nash's (1953) demand game. In the demand game, two players make simultaneous demands and each one receives the payoff she requests if both payoffs are jointly feasible and nothing otherwise. Our model is more complex since strategies are not unidimensional and the threat point is decided endogenously. Fig. 1 illustrates this in a bargaining situation with three alternatives, each one represented at the corresponding vertex of the simplex. Fig. 1a depicts the non-consensual and sincere strategy profile (s_1, s_2) while Fig. 1b shows the consensual and sincere strategy profile (s'_1, s'_2). Under (s_1, s_2), player 1 (resp. player 2) approves every lottery in the closed subset labeled s_1 (resp. s_2). The outcome induced by (s_1, s_2) is the uniform probability measure over $s_1 \cup s_2$ and the expectation of such a measure is the barycenter $b(s_1 \cup s_2)$. In Fig. 1b, the strategies s'_1 and s'_2 intersect so that the induced outcome is $b(s'_1 \cap s'_2)$. Note that either players can deviate to some non-consensual strategy that induces an outcome arbitrarily close to $b(s'_1 \cup s'_2)$. Hence, $b(s'_1 \cup s'_2)$ is the endogenous threat point that sustains the equilibrium outcome $b(s'_1 \cap s'_2)$.

This example suggests that, under a non-consensual strategy profile, players have two joint incentives: (1) approving a large subset of lotteries so that the induced expected outcome is as close as possible to it and, consequently, (2) playing some sincere strategy that approves every lottery in the upper contour set of some indifference curve. These two incentives work together so that both players approve bigger and bigger sets. The consequence is that a non-consensual strategy profile cannot be an equilibrium. In Section 3, we prove that every equilibrium strategy profile has a nonempty intersection in the same way as in Fig. 1b. Finally, note that in this figure either player can deviate to a non-sincere consensual strategy and still induce outcome $b(s'_1 \cap s'_2)$ as long as the resulting intersection is also $b(s'_1 \cap s'_2)$. But partial honesty guarantees that players would rather play the sincere strategy.

Building on these observations, we prove that the approval bargaining game has the following properties.

(1) *Existence of equilibrium*: Every game has an equilibrium in sincere strategies.
(2) *Consensual equilibria*: In every equilibrium, players agree on some subset of lotteries.
(3) *Sincerity and Pareto efficiency*: If players are partially honest, every equilibrium outcome is in sincere strategies and Pareto efficient.

[2] Thus, sincerity in this context implies the sincerity notion used in the approval voting literature in which a strategy is *sincere* if, whenever it contains an alternative, it also contains the other alternatives that the player prefers to it. See Merill and Nagel (1987), Brams (2008), and Núñez (2014) for works dealing with sincerity under approval voting.

[3] Núñez (2015) analyzes a voting rule in this fashion and shows that it leads to type-revelation with many voters.

[4] The difference is that in Babichenko and Schulman (2015) and Núñez and Laslier (2015) agents approve alternatives while, in the current model, agents approve lotteries over alternatives. This is the main driving force behind of the efficiency properties of our model.