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Bayesian persuasion with multiple senders and rich signal spaces

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ABSTRACT

A number of senders with no *ex ante* private information publicly choose signals whose realizations they observe privately. Senders then convey verifiable messages about their signal realizations to a receiver who takes a non-contractible action that affects the welfare of all players. The space of available signals includes all conditional distributions of signal realizations and allows any sender to choose a signal that is arbitrarily correlated with signals of others. We characterize the information revealed in pure-strategy equilibria and show that greater competition tends to increase the amount of information revealed.

1. Introduction

In Kamenica and Gentzkow (2011), we study the problem of "Bayesian persuasion" where a single sender chooses what information to gather and communicate to a receiver who then takes a non-contractible action that affects the welfare of both parties. In this paper, we extend the analysis to multiple senders.

Consider the example of a prosecutor and a defense attorney (senders) arguing a case before a judge (Receiver). The are two states of the world: the defendant is either *guilty* or *innocent*. The judge must choose one of two actions: to *acquit* or *convict* the defendant. The judge gets utility 1 for choosing the just action (convict when guilty or acquit when innocent) and utility 0 for choosing the unjust action. The prosecutor gets utility 1 if the judge convicts and 0 otherwise. The defense gets utility 1 if the judge acquits and 0 otherwise. All share a prior belief that $\Pr(guilty) = 0.3$.

The attorneys simultaneously choose investigations to conduct and are required by law to report the full outcome of their investigation to the judge.[1] We think of the choice of investigation as capturing decisions such as which witnesses to call, which questions to ask, which documents to subpoena, and so on. Formally, investigations are distributions of signal realizations conditional on the state. The noise in these signals may be correlated, for example because the attorneys choose to call the same witness.

Kamenica and Gentzkow (2011) consider a version of this game where there is only one sender, the prosecutor. They show that the optimal signal for the prosecutor in that case leaves the judge with two possible posterior beliefs: either $Pr(guilty) = 0$, in which case she acquits, or $Pr(guilty) = 0.5$, in which case she is indifferent but breaks ties in the prosecutor's favor.

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[1] The model we consider below considers situations where senders observe the outcome privately and then convey verifiable messages about it, but this does not change the equilibrium outcome (cf.: Gentzkow and Kamenica, 2017b).

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To see what happens in the full two-sender game, consider a proposed equilibrium in which the prosecutor chooses this unilaterally optimal signal. Suppose the defense attorney can choose a signal arbitrarily correlated with that of the prosecutor—i.e., she can separately choose how much additional information to reveal at each realization of the prosecutor's signal. Then, the defense attorney's best response is clearly to reveal additional information when the prosecutor's signal suggests guilt: at this point, the judge is convicting for sure, and even very weak evidence in favor of innocence is enough to change the outcome. Any informative signal would thus increase the defense attorney's payoffs. Were the defense attorney to play such a best response, however, the prosecutor would in turn prefer to reveal additional information at any profile of signal realizations where the judge was acquitting the defendant and had an interior belief. Iterating this logic, the unique equilibrium is for both parties to choose full revelation. (We show this formally in Section 6.)

This example suggests two key ideas which we develop in more detail. First, any belief that occurs in an equilibrium must be stable in the sense that if Receiver holds this belief neither sender would want to reveal additional information. In the trial example, the only such beliefs are $Pr(guilty) = 0$ and $Pr(guilty) = 1$. We show that such beliefs are in general easy to identify using the geometric approach developed by Aumann and Maschler (1995) and Kamenica and Gentzkow (2011). Moreover, we show that this stability condition is not only necessary but sufficient for equilibrium. This sufficiency result provides a simple algorithm for characterizing the complete set of (pure-strategy) equilibrium outcomes.

Second, in the aforementioned example, competition increases the extent of information revelation: the game between a prosecutor and a defense attorney generates more information than either player would reveal on her own. We show that this result extends to situations with arbitrary preferences and that it extends to various notions of what it means for an environment to be "more competitive."

In our general model, several senders, who have no *ex ante* private information, simultaneously select costless signals about an unknown state of the world. The set of possible signals is rich; it includes all conditional distributions of signal realizations given the state, and it allows arbitrary correlation among the senders' signals. To formalize this signal space, we draw on the approach in Green and Stokey (1978).

Each sender privately observes his signal realization and then sends a verifiable message about it to Receiver. The assumption of privately observed signal realizations allows for the possibility that senders have the option to hide unfavorable information *ex post*—a generalization beyond the motivating example and the baseline case in Kamenica and Gentzkow (2011). Receiver observes what experiments were conducted and the messages about their outcomes sent by the senders. She then takes a non-contractible action that affects the welfare of all players. The state space is finite. Receiver and each of the senders have arbitrary, state-dependent, utility functions over the Receiver's action and the state of the world. Throughout the paper we focus on pure-strategy equilibria of the game.[2]

The information revealed in an equilibrium of this game can be succinctly summarized by the distribution of Receiver's posterior beliefs (Blackwell, 1953). We refer to such a distribution as an *outcome* of the game and order outcomes by informativeness according to the usual Blackwell criterion.

We begin our analysis by establishing the following lemma: if the senders other than i together induce some outcome τ', sender i can unilaterally deviate to induce some other outcome τ if and only if τ is more informative than τ'. The "only if" part of this lemma is trivial and captures a basic property of information: an individual sender can unilaterally increase the amount of information revealed, but can never decrease it below the informational content of the other senders' signals. The "if" part of the lemma is more substantive, and draws on the assumption that senders have access to a rich set of possible signals. Our lemma implies that no outcome can be a pure-strategy equilibrium if there exists a more informative outcome preferred by any sender. This property is the fundamental reason why competition tends to increase information revelation in our model.

Our main characterization result provides an algorithm for finding the full set of pure-strategy equilibrium outcomes. We consider each sender i's value function over Receiver's beliefs $\hat{v}_i$ and its concavification V_i (the smallest concave function everywhere above $\hat{v}_i$). Kamenica and Gentzkow (2011) show that when there is a single sender $i = 1$, any belief μ that Receiver holds in equilibrium must satisfy $\hat{v}_1(\mu) = V_1(\mu)$. We extend this result and establish that, when there are two or more senders, a distribution of posteriors is an equilibrium outcome *if and only if* every belief μ in its support satisfies $\hat{v}_i(\mu) = V_i(\mu)$ for all i. Identifying the set of these "coincident" beliefs for a given sender is typically straightforward. Any given outcome is an equilibrium if and only if its support lies in the intersection of these sets.

We then turn to the impact of competition on information revelation. We consider three ways of increasing competition among senders: (i) moving from collusive to non-cooperative play, (ii) introducing additional senders, and (iii) decreasing the alignment of senders' preferences. Since there are typically many equilibrium outcomes, we state these results in terms of set comparisons that modify the strong and the weak set orders introduced by Topkis (1978). We show that, for all three notions of increasing competition, more competition never makes the set of outcomes less informative (under either order). We also show that if the game is zero-sum for any subset of senders, full revelation is the unique equilibrium outcome whenever the value functions are sufficiently nonlinear.

[2] In Section 4, we briefly discuss the complications that arise with mixed strategies.