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Note

Every random choice rule is backwards-induction rationalizable ☆

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ABSTRACT

Motivated by the literature on random choice and in particular the random utility models, we extend the analysis in Bossert and Sprumont (2013) to include the possibility that players exhibit stochastic preferences over alternatives. We prove that every random choice rule is backwards-induction rationalizable.

1. Introduction

Bossert and Sprumont (2013) define a choice function as backwards-induction rationalizable "if there exists a finite perfect-information extensive-form game such that for each subset of alternatives, the backwards-induction outcome of the restriction of the game to that subset of alternatives coincides with the choice from that subset." Bossert and Sprumont (2013) then prove that every choice function is backwards-induction rationalizable. They focus on games where all players have *strict preferences* over the alternatives.

It is well known that individual choices exhibit variability, in both experimental and market settings; see for example, Sippel (1997), McFadden (2001), and Manzini et al. (2010). The theoretical literature on random choice has focused largely on interpreting random choice as random utility maximization.[1] Motivated by the literature on random choice and in particular the random utility models (Block and Marschak, 1960), we extend the analysis in Bossert and Sprumont (2013) to include the possibility that players exhibit *stochastic preferences* over alternatives.

In a collective decision-making setting, if some player has a stochastic preference, then not surprisingly, the collective actions of the players might lead to a random outcome. We study the testable aspects of collective decision-making, allowing

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[1] A random utility model is described by a probability measure over preference orderings, and the player selects the maximal alternative available according to the randomly assigned preference ordering; see for example, the seminal work of Block and Marschak (1960).

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for *stochastic preferences* of the players. We extend the Bossert–Sprumont theorem, and prove that every random choice rule is backwards-induction rationalizable via stochastic preferences.

This note contributes to the emerging literature that applies the revealed preference approach to the study of collective decisions. Yanovskaya (1980), Sprumont (2000) and Galambos (2005) consider choice correspondences and Nash equilibria of normal-form games. Ray and Zhou (2001) and Ray and Snyder (2013) study Nash equilibria and sub-game perfect equilibria on extensive-form games. Xu and Zhou (2007) and Bossert and Sprumont (2013) examine when choice functions can be rationalized by an extensive-form game. Rehbeck (2014) and Xiong (2014) extend the Bossert–Sprumont theorem, and prove that every choice correspondence is backwards-induction rationalizable via *weak preferences*. In particular, the construction of the extensive-form game hinges upon a player who exhibits complete indifference among all alternatives.

2. Definitions

Let X be a given finite universal set of alternatives, and denote by $\mathcal{P}(X)$ the collection of all nonempty subsets of X. The elements of $\mathcal{P}(X)$ are viewed as feasible sets that the players collectively choose an alternative from. We use $A, B, C, \ldots$ to denote alternative sets, and $x, y, z, \ldots$ to denote alternatives. Throughout the rest of the paper, unless it leads to confusion, we abuse the notation by suppressing the set delimiters, e.g., writing x rather than $\{x\}$. We use the following notational convention: $xy := x \cup y$.

A *choice function* is a map $f : \mathcal{P}(X) \to X$ such that $f(A) \in A$ for all $A \in \mathcal{P}(X)$. A *random choice rule* is a map $\rho : X \times \mathcal{P}(X) \to [0, 1]$ such that for all $A \in \mathcal{P}(X)$, we have i) $\sum_{x \in A} \rho(x, A) = 1$; and ii) $\rho(x, A) = 0$ for all $x \notin A$. The interpretation is that $\rho(x, A)$ denotes the probability that alternative x is chosen when the possible alternatives faced by the players are the alternatives in A.

In what follows, we present the relevant definitions and notations. Whenever possible, we keep the notations consistent with Bossert and Sprumont (2013) and Rehbeck (2014). We suggest that readers familiar with these two papers skip this section and return to it as needed.

Preference ordering. A *preference ordering* is a reflexive, complete, transitive and antisymmetric binary relation. We denote by $\mathcal{R}_A$ the set of all preference orderings on $A \in \mathcal{P}(X)$.

Precedence relation. Let $\prec$ be a transitive and asymmetric binary relation on a nonempty and finite set N. We say that $n \in N$ is a *direct predecessor* of $n' \in N$ if $n \prec n'$ and there is no $n'' \in N$ such that $n \prec n'' \prec n'$. Similarly, we say that $n \in N$ is a *direct successor* of $n' \in N$ if $n' \prec n$ and there is no $n'' \in N$ such that $n' \prec n'' \prec n$. The set of direct predecessors of $n \in N$ is denoted by $P(n)$. The set of direct successors of $n \in N$ is denoted by $S(n)$.

Tree. A *tree* Γ is given by a quadruple $(0, D, T, \prec)$, where the variables are defined as follows:

(i) the notation 0 is the *root*;
(ii) the variable D is a finite set of *decision nodes* such that $0 \in D$;
(iii) the variable T is a nonempty and finite set of *terminal nodes* such that $D \cap T = \emptyset$;
(iv) the notation $\prec$ is a transitive and asymmetric *precedence relation* on the set of all nodes $N = D \cup T$ such that:
  (a) $P(0) = \emptyset$, and $|S(0)| \geq 1$;
  (b) for all $n \in D \setminus \{0\}$, $|P(n)| = 1$, and $|S(n)| \geq 1$;
  (c) for all $n \in T$, $|P(n)| = 1$, and $S(n) = \emptyset$.

Path. A *path in* Γ from a decision node $n \in D$ to a terminal node $n' \in T$ (of length $K \in \mathbb{N}$) is an ordered $(K+1)$ tuple $(n_0, n_1, \ldots, n_K) \in N^{|K+1|}$ such that $n_0 = n$, $\{n_{k-1}\} = P(n_k)$ for all $k \in \{1, 2, \ldots, K\}$, and $n_K = n'$.

Game. A *game* is a triple $G = (\Gamma, g, \pi)$ where

(i) $\Gamma = (0, D, T, \prec)$ is a tree;
(ii) $g : T \to X$ is an *outcome function* that maps each terminal node $n \in T$ to an alternative $g(n) \in X$;
(iii) π is a probability measure over the space of *preference assignment maps*, where each preference assignment map $R : D \to \mathcal{R}_X$ specifies for each decision node $n \in D$ a preference ordering $R(n) \in \mathcal{R}_X$. We denote by $\mathfrak{R}_{D,X}$ the space of all such preference assignment maps, and denote by $\Delta\mathfrak{R}_{D,X}$ the set of all probability measures over $\mathfrak{R}_{D,X}$. Formally, $\pi \in \Delta\mathfrak{R}_{D,X}$.

We focus on games in which the uncertainty on R resolves before any player makes a move, and the realization of R is commonly known among all the players. Let δ_R denote the degenerate measure at the preference assignment map R. For simplicity, sometimes we write $G = (\Gamma, g, R)$ rather than $G = (\Gamma, g, \delta_R)$.

Restriction of game. Fix a game $G = (\Gamma, g, \pi)$, we define the *restriction* of game G on $A \in \mathcal{P}(X)$ as $G|A = G_A = (\Gamma_A, g_A, \pi_A)$, where