Games and Economic Behavior 104 (2017) 666–673

Contents lists available at ScienceDirect

Games and Economic Behavior

www.elsevier.com/locate/geb

Note

Abraham Wald's complete class theorem and Knightian uncertainty

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ARTICLE INFO

Article history:
Received 5 March 2014
Available online 5 July 2017

JEL classification:
C72
C81
C90
D01
D03
D81

Keywords:
Ambiguity
Decision theory
Knightian uncertainty
Experimental design

ABSTRACT

I study the implications of Wald's (1947) complete class theorem for decision making under Knightian uncertainty (or ambiguity). Suppose we call someone who uses Wald's approach to statistical decision making a Waldian. A Waldian may then have preferences over acts that are not in agreement with subjective expected utility but always chooses as if she was a subjective expected utility maximizer. In particular, even Wald's (1945) minmax decision rule is consistent with subjective expected utility.

"Everything has been said but not everyone has said it."
[Morris Udall, 1922–1992, US American Politician]

"Es ist schon alles gesagt, nur noch nicht von allen."
[Karl Valentin, 1882–1948, Bavarian Comedian]

1. Introduction

Ellsberg (1961) conducted a thought experiment, asking individuals about their preferences over potential choices in his famous two- and three-color urn decision problems. The answers individuals gave to these questions demonstrate that many individuals have preferences that are inconsistent with the subjective expected utility (SEU) models of Savage (1954)

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[1] The three previous versions of this paper were circulated under the title "Inferring preferences from choices under uncertainty", "An alternative subjective expected utility representation theorem", and "A rational ambiguity averse person will never display her ambiguity aversion" respectively, and are still available as SSRN working papers. I would like to thank seminar participants at the Universities of Bielefeld, Bristol, Exeter, Heidelberg, Oslo, and Simon Fraser University as well as Herbert Dawid, Christoph Diehl, Jürgen Eichberger, David Freeman, Michael Greinecker, Faruk Gul, Chiaki Hara, David Kelsey, Peter Klibanoff, Sujoi Mukerji, Jörg Oechssler, Phil Reny, Frank Riedel, Arthur Robson, Zvi Safra, and particularly the anonymous associate editor and his or her assistant as well as the anonymous referees at Games and Economic Behavior for many helpful comments and suggestions. I am also particularly grateful to Jonathan Weinstein who pointed me to the work of Abraham Wald.

http://dx.doi.org/10.1016/j.geb.2017.06.012
0899-8256/

and Anscombe and Aumann (1963).[2] There is now a large literature, beginning at least with Gilboa and Schmeidler (1989), including also Klibanoff et al. (2005), Maccheroni et al. (2006), Seo (2009), and Cerreia-Vioglio et al. (2013) in which an axiomatic foundation for such non-SEU preferences is given.

Note that this is all about preferences. It is not immediately clear that this is also about choices. In a related problem, that of eliciting preferences from choices for models that violate the von Neumann and Morgenstern (1944) expected utility axioms for objective lotteries, we know, from Holt (1986), Karni and Safra (1987), and Segal (1988) that certain experiments such as those using the Becker et al. (1964) mechanism do not generally allow the correct elicitation of non-expected utility preferences. This literature, thus, demonstrates that eliciting preferences is not straightforward when the decision maker has preferences that deviate from standard expected utility.

In this paper, I discuss whether it is possible to elicit preferences that do not agree with SEU. I do this under the assumption, also made in e.g. Gilboa and Schmeidler (1989), Klibanoff et al. (2005), Maccheroni et al. (2006), and Seo (2009), that the decision maker evaluates objective lotteries according to expected utility.

As I am interested in individuals' choices and the elicitation of their preferences, I need to discuss three additional assumptions that are not directly related to preferences. First, I assume that individuals can actively and objectively randomize when making their choices.[3] Second, I assume that individuals can commit to following the realization of their random choice in case they choose randomly. Third, I assume that, when faced with more than one decision problem, individuals form a global plan (a detailed plan specifying a choice in every subproblem). These assumptions are not made because of their empirical plausibility but because of their normative appeal.[4] I discuss their role for the result and their plausibility in the discussion section of this paper.

To give a preview of the discussion below, note that there are two distinct issues that Ellsberg's (1961) thought experiments raise. To see these, consider a slight variation of the two-color Ellsberg (1961) urn thought experiment.[5]

There are two urns. One urn, the risky or unambiguous urn, holds 49 green and 51 black balls. The other urn, the ambiguous urn, holds 100 balls, all of which are either red or white, but the exact composition is not known. Consider the following three bets. In bet zero, the decision maker (DM) receives a monetary prize if a (uniformly) randomly drawn ball from the risky urn is green and receives nothing otherwise. In bet one, the DM receives the same prize if a (uniformly) randomly drawn ball from the ambiguous urn is red and receives nothing otherwise. In bet two, finally, the DM receives the same prize if a (uniformly) randomly drawn ball from the ambiguous urn is white and receives nothing otherwise.

Suppose we ask the DM how she would choose if she were presented with a choice between bets zero and one only. She might state that she prefers bet zero. Suppose we ask her then how she would choose if she were presented with a choice between bet zero and bet two. She might state that she prefers bet zero.

These are hypothetical decision problems. What would happen if we give her an actual choice between various bets? Suppose first, and this is issue number one, we ask her to choose among all three bets. After she chooses we then perform the drawing of balls and pay her accordingly. Will she choose bet zero? And if she does, what does this tell us about her?

Raiffa (1961) provided the following argument that she "should" not choose bet zero. She should consider choosing objectively randomly by flipping a fair coin. If the coin comes up heads she should choose bet one, if it comes up tails she should choose bet two. Then, regardless of the color of the ball drawn from the ambiguous urn, she has a probability of 1/2 of winning the prize, while bet zero only gives her a 49/100 probability of winning.

In the language of game theory, the random bet (1/2 on each of the two bets one and two) strictly dominates bet zero, as it provides a strictly higher winning probability (and thus expected utility, provided the DM values the prize more than receiving nothing) than bet zero in every possible state of nature (i.e., for any possible composition of the ambiguous urn). If the DM avoids dominated strategies she will not choose bet zero, when presented with the three bets, and will thus not fully reveal her "preferences". That is, she does not reveal that she would have chosen bet zero if she were only presented with bets zero and one, and bet zero again if she were only presented with bets zero and two.

Giving the DM just one decision problem with the three bets is, however, only one way to try to elicit the DM's preferences. Suppose, and this is issue number two, we give her two decision problems. We ask her to choose one of bets zero and one, and one of bets zero and two. We then need to specify how exactly we pay the DM in this case. We have many options. I will explore only one here. For further possibilities see the more general treatment in Section 3. Suppose we ask her to make her choices in both problems, then we choose one of the two problems objectively randomly and equally likely with the understanding that this decision problem is then used to pay her. Note that now the DM does not even have to randomize herself. The choice of bet one in problem one and bet two in problem two gives her a winning probability of 1/2

[2] Note that there are objective probability distributions in the Ellsberg (1961) experiment, in the form of the risky urn and the uniformly random draws of balls from both urns. While objective probabilities are present in the Anscombe and Aumann (1963) model, the Savage (1954) model strictly speaking has no objective uncertainty.

[3] Note that Gilboa and Schmeidler (1989), Klibanoff et al. (2005), and Maccheroni et al. (2006) each provide an axiomatization of preferences over acts only, not over the set of objective randomizations over acts. Battigalli et al. (2017) provide a unifying framework in which the above-mentioned preference models can be embedded and that allows for objective randomizations over acts.

[4] One could probably argue that the literature on ambiguity aversion at least implicitly rejects some of these three assumptions. Empirically this rejection may well be warranted.

[5] The variation is used to avoid payoff-ties.