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Fast multiple inversion for stress analysis from fault-slip data

Katsushi Sato

Division of Earth and Planetary Sciences, Kyoto University, Kyoto 606-8502, Japan

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ABSTRACT

The multiple inverse method is widely used to invert multiple stress tensors from fault-slip data caused by polyphase tectonics. A practical problem of the method is the time-consuming computation related to its iterative procedure. This paper describes a way of accelerating the computation by replacing an exhaustive grid search for the optimal stress tensor by direct calculation using an analytical solution. Furthermore, a technique to reduce noise in the result was developed based on the estimation of instabilities of solutions.

1. Introduction

Stress tensor inversion methods are widely used to infer tectonic stress state from fault-slip data. Input fault data are collected from geological outcrops, seismic focal mechanisms, rock core samples, and underground images obtained by three-dimensional seismic surveys. Among the variety of methods the multiple inverse method (Yamaji, 2000), hereafter abbreviated as MIM, has an advantage in separating multiple stress tensors from a mixture of geological faults yielded from spatial or temporal change of tectonic stress state. This method has been used by many researchers in various regions (e.g., Yamada and Yamaji, 2002; Yamaji, 2003; Sippel et al., 2009; Chan et al., 2010) and further methodological improvement is now ongoing. MIM has been extended to analyze seismic focal mechanisms without a priori specification of fault planes from paired orthogonal nodal planes (Otsubo et al., 2008), improved to objectively recognize multiple solutions by means of clustering techniques (Otsubo and Yamaji, 2006), and enhanced in its resolution through development of uniform computational grid (Sato and Yamaji, 2006b; Yamaji and Sato, in press).

A fault-slip data set is described as heterogeneous when it includes faults caused by different stresses. A conventional method of stress inversion (e.g., Angelier, 1979) determines an optimal stress tensor for a whole data set, though the solution is meaningless if the data set is heterogeneous. MIM can detect multiple stress tensors through an iterative sampling procedure. When a data set has N faults, MIM extracts a subset including k faults from it and determines an optimal stress tensor for the subset by exhaustive grid search. This process is repeated ${}_NC_k$ times for all possible combinations of k-element subsets. A great number of stress tensors are obtained and their concentrations are interpreted as desired tectonic stresses (Fig. 1). This iterative calculation also has an effect of enhancing solutions from natural noisy fault-slip data.

A problem of MIM lies in its computational cost. It takes between a few hours and several days to analyze several hundred to a thousand faults by a personal computer. The cost is proportional to the number of fault subsets ${}_NC_k$, which is order of $O(N^k)$ by Landau's symbol. The number of faults in a subset k is empirically set to four or five (Yamaji, 2000). Therefore the cost is $O(N^4)$ or $O(N^5)$. This fact generally limits the total number of faults N up to a thousand.

Each determination of optimal stress for fault subsets is done by exhaustive grid search on 60,000 uniformly spaced stress tensors (Sato and Yamaji, 2006b) by default. This study proposes a direct algorithm for determination of an optimal stress tensor. Although the new technique is applicable only to four-element subsets, it calculates the numerous stress solutions several times faster than conventional MIM. A method of noise reduction by estimating instabilities of solutions is also provided.

2. Method

2.1. Wallace–Bott hypothesis

MIM as well as recent stress tensor inversion techniques is based on an assumption that a fault slips in the direction of shear

E-mail address: k_sato@kueps.kyoto-u.ac.jp

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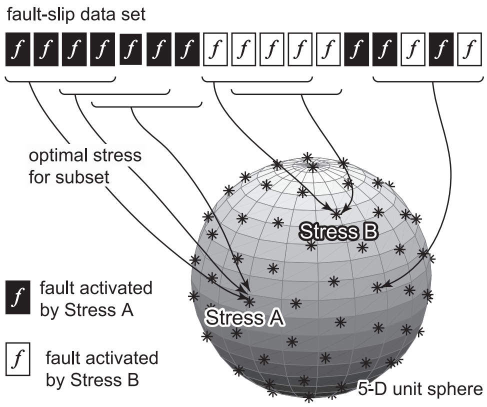

Fig. 1. Schematic figure illustrating the procedure of the multiple inverse method (MIM) of detecting multiple stress tensors from a heterogeneous fault-slip data set. The data set is a mixture of black and white f symbols representing faults activated by different stresses A and B, respectively. MIM extracts subsets of four or five faults from whole data and determines optimal solutions for them by means of exhaustive grid search on the deviatoric stress space (Sato and Yamaji, 2006b), which is geometrically the surface of a five-dimensional unit sphere. Homogeneous subsets are expected to concentrate their votes at the grid points corresponding to stresses A or B, while the meaningless solutions from heterogeneous subsets should be placed randomly.

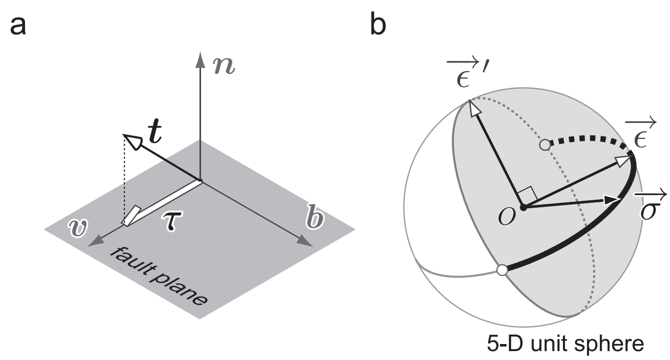

Fig. 2. Wallace–Bott hypothesis as the principle of stress tensor inversion. The slip direction of a fault is assumed to coincide with the shear stress direction exerted by the tectonic stress in question. (a) In the physical space, observable fault parameters are represented by unit vectors $\boldsymbol{v}$, $\boldsymbol{b}$, and $\boldsymbol{n}$. A correct stress tensor gives shear stress vector $\boldsymbol{\tau}$, which is the projection of traction vector $\boldsymbol{t}$ onto fault plane, in the direction of slip $\boldsymbol{v}$. (b) Schematic figure of deviatoric stress space. The Wallace–Bott hypothesis is geometrically expressed as the constraint on the stress tensor represented by $\vec{\sigma}$ from a fault-slip datum. The fault parameters $\vec{\epsilon}$ and $\vec{\epsilon}\,'$ specify a half great circle called the Fry arc (bold line) on which the $\vec{\sigma}$ vector is required to lie.

stress, which is called Wallace–Bott hypothesis (Wallace, 1951; Bott, 1959, illustrated in Fig. 2a). Input data for stress inversion analysis, called fault-slip data, contain fault plane orientations, slip orientations, and shear senses, while the unknown parameters are described by stress tensors. The direction of shear stress on a fault plane depends on four of the six independent components of the stress tensor. Let $\boldsymbol{\sigma}$, whose components are denoted by σ_{ij} (i=1 to 3, j=1 to 3), be a reduced stress tensor with four degrees of freedom. Two normalization conditions imposed on $\boldsymbol{\sigma}$ can be freely chosen. The first and second invariants are normalized in this study; i.e.,

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 = 0 \tag{1}$$

and

$$J_2 = -\sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1 = 1, \tag{2}$$

where σ_1, σ_2, and σ_3 are the principal stress magnitudes ($\sigma_1 \geq \sigma_2 \geq \sigma_3$, compression is positive). Let $\boldsymbol{n} = (n_1,n_2,n_3)^{\mathrm{T}}$ and $\boldsymbol{v} = (v_1,v_2,v_3)^{\mathrm{T}}$ be the unit vectors in the directions of the fault normal and slip direction, respectively. The superscript T denotes the transpose of a vector or a matrix. Hereafter all vectors are column vectors. Cauchy's formula gives the traction vector exerted on a fault plane by a stress as $\boldsymbol{t} = \boldsymbol{\sigma n}$. The shear stress is derived by projecting $\boldsymbol{t}$ onto the fault plane as $\boldsymbol{\tau} = \boldsymbol{t} - \boldsymbol{n}\boldsymbol{n}^{\mathrm{T}}\boldsymbol{t}$. The Wallace–Bott hypothesis requires $\boldsymbol{\tau}$ to be in the same direction as $\boldsymbol{v}$.

Fry (1999) decomposed the Wallace–Bott condition into

$$\boldsymbol{b} \cdot \boldsymbol{t} = 0 \tag{3}$$

and

$$\boldsymbol{v} \cdot \boldsymbol{t} > 0, \tag{4}$$

where the unit vector $\boldsymbol{b} = \boldsymbol{n} \times \boldsymbol{v}$ is perpendicular to both $\boldsymbol{n}$ and $\boldsymbol{v}$. Eq. (3) requires the shear stress vector $\boldsymbol{\tau}$ to be parallel to the observed slip direction $\boldsymbol{v}$, while Eq. (4) represents the correspondence of shear sense (Fig. 2a). Sato and Yamaji (2006a) introduced the deviatoric stress space into stress inversion analysis, in which reduced stress tensors and fault-slip data are represented by five-dimensional unit vectors (Fig. 2b). They reformulated Eqs. (3) and (4) as

$$\vec{\epsilon}\,' \cdot \vec{\sigma} = 0 \tag{5}$$

and

$$\vec{\epsilon} \cdot \vec{\sigma} > 0, \tag{6}$$

respectively. The vectors in Eqs. (5) and (6) are defined as

$$\vec{\sigma} = \begin{pmatrix} \sigma_{11}/\sqrt{2} \\ \sigma_{22}/\sqrt{2} \\ \sigma_{33}/\sqrt{2} \\ \sigma_{23} \\ \sigma_{31} \\ \sigma_{12} \end{pmatrix}, \quad \vec{\epsilon}\,' = \begin{pmatrix} \sqrt{2}b_1n_1 \\ \sqrt{2}b_2n_2 \\ \sqrt{2}b_2n_2 \\ b_2n_3+b_3n_2 \\ b_3n_1+b_1n_3 \\ b_1n_2+b_2n_1 \end{pmatrix}, \quad \vec{\epsilon} = \begin{pmatrix} \sqrt{2}v_1n_1 \\ \sqrt{2}v_2n_2 \\ \sqrt{2}v_2n_2 \\ v_2n_3+v_3n_2 \\ v_3n_1+v_1n_3 \\ v_1n_2+v_2n_1 \end{pmatrix}. \tag{7}$$

The normalization conditions of the stress tensor (Eqs. (1) and (2)) and the orthogonality of unit vectors representing fault parameters (Fig. 2a) imply

$$\sigma_{11} + \sigma_{22} + \sigma_{33} = \epsilon'_1 + \epsilon'_2 + \epsilon'_3 = \epsilon_1 + \epsilon_2 + \epsilon_3 = 0, \tag{8}$$

$$|\vec{\sigma}| = |\vec{\epsilon}\,'| = |\vec{\epsilon}| = 1, \tag{9}$$

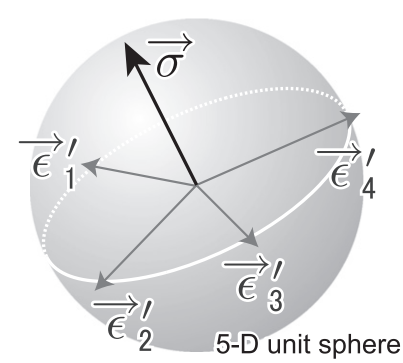

Fig. 3. Schematic figure illustrating how to calculate the direct solution of stress tensor inversion. When we have four fault-slip data, four $\vec{\epsilon}\,'$ vectors are specified in the five-dimensional deviatoric stress space. The parallel conditions between fault-slip directions and shear stress vectors require the $\vec{\sigma}$ vector representing the stress tensor to be perpendicular to all four $\vec{\epsilon}\,'$ vectors. The analytical solution to this even-determined problem can be uniquely obtained as the direction of the five-dimensional cross product of $\vec{\epsilon}\,'$ vectors. Note that four $\vec{\epsilon}\,'$ vectors must be linearly independent in the five-dimensional space, although this schematic figure looks as if they were two-dimensionally coplanar owing to lack of dimension. The white circle spanned by them represents not a two-dimensional circle but a four-dimensional space.